AN INVESTIGATION OF NUCLEAR EXCURSIONS TO DETERMINE THE SELF-SHUTDOWN EFFECTS IN THERMAL, HETEROGENEOUS, HIGHLY ENRICHED, LIQUID-MODERATED REACTORS

by

JOHN ROBERT FAGAN

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Nomenclature of Terms Not Defined in Text

k_{eff}	Effective multiplication factor.
ε	Fast fission factor.
p	Resonance escape probability.
η	Neutrons born per thermal absorption in fuel.
f	Thermal utilization.
B^2	Buckling, cm^{-2}.
L	Thermal diffusion length, cm.
κ	$1/L$, cm^{-1}.
τ	Fermi age, cm^2.
v	Neutron velocity, cm/sec.
q_{oo}	Heat generation rate in center of fuel at start of pulse, $btu/hrft^3$.
τ	Reciprocal period of power rise, sec^{-1}.
A_j, λ_j	Parameters for empirical fit of heat generation rate, $btu/hrft^3$ and sec^{-1} respectively.
B_i, β_i	Parameters for empirical fit of fuel plate surface boundary condition, ^oF and sec^{-1} respectively.
L, L_1	Thickness of fuel and moderator, respectively in slab geometry, cm.
R, R_1	Thickness of fuel and moderator, respectively in cylindrical geometry, cm.
x	Distance in fuel from the center of unit cell, slab geometry, cm.
x_1	Distance in moderator from outside of unit cell, slab geometry, cm.
r	Distance in fuel and moderator from center of unit cell, cylindrical geometry, cm.
i, j, n	Summation indices.

f,m	Subscripts denoting fuel and moderator, respectively.
p,s	No. of terms in heat generation rate and interface boundary condition approximations.

1.0 INTRODUCTION

The concept of reactor safety is extremely important in the engineering application of nuclear power systems. The United States Atomic Energy Commission has therefore authorized an extensive study in this area. This investigation uses experimental data resulting from that study to attempt to define the mechanisms of reactor shutdown.

The safety of a nuclear reactor system is usually considered in terms of its void and temperature coefficients of reactivity. If one designs a reactor in such a manner as to make both of these coefficients negative, the system will tend to stabilize itself if some external perturbation is placed on the system. This is due to the fact that when the excess reactivity, $k_{eff}-1$, of a system is increased, the power tends to rise, thereby increasing the temperature of the system, and this in turn causes a decrease in reactivity. In the case of a liquid moderated system, voids may be introduced which will further decrease the reactivity.

If one considers the bare reactor age-diffusion criticality equation, $k_{eff} = \frac{\varepsilon p \eta f \, e^{-B^2 \tau}}{1 + L^2 B^2}$, it is possible to see how these effects manifest themselves in the nuclear constants (22). In the case of the void coefficient, the significant effect is on the Fermi age, τ. Since part of the moderator is removed, the age increases, thus decreasing the fast non-leakage factor, $e^{-B^2 \tau}$. In the case of the temperature coefficient there are several effects that must be considered. First, the moderator expands because of the positive coefficient of expansion of the liquid.

This results in a decrease in the density of the moderator, and if the reactor is under-moderated the fast non-leakage factor and p will decrease. Thus it is evident that one safety criteria is that the core should always be slightly under-moderated even though this will increase the critical mass. Second, the fuel elements expand, expelling more of the moderator from the core causing k_{eff} to decrease; however, simultaneously, the effective size of the core increases, causing a decrease in the buckling, B^2, of the system. This decrease in buckling has both a positive and a negative effect upon reactivity or k_{eff}. The increase in size increases the non-leakage factor for both fast and thermal leakage, $e^{-B^2\tau}$ and $1/(1 + L^2B^2)$, respectively. The removal of moderator increases τ reducing reactivity as described in the void coefficient discussion. All of the effects described above with the exception of the buckling, must be considered to be of a delayed nature.

Another group of effects exists which affect the reactivity immediately and these effects are therefore classified as prompt (41). The first of these prompt effects is Doppler broadening (38). Because of the increased kinetic energy of U^{238} target nuclei with increased temperature, the width of resonance absorption is increased, but the height of the peak is decreased (22), because the total area beneath the resonance curve remains constant If the resonance absorption cross sections are large, so that essentially all neutrons with energies in the resonance region are captured, the widening of the region will result in a decrease in the resonance escape probability, p, and thus the reactivity decreases as the temperature rises. A second prompt effect is caused by hardening of the thermal neutron spectrum as the moderator temperature

rises. The result of this hardened spectrum is that the average thermal-neutron velocity increases, L^2 increases and the thermal non-leakage factor, $\frac{1}{1 + L^2B^2}$, decreases. One would expect that the neutrons per absorption would also be affected by this spectral change but assuming the normal 1/v dependence for all cross sections, this effect cancels since the terms comprising this effect are composed of the ratios of cross sections. One final effect which is neither part of the temperature coefficient nor the void coefficient must be considered. This is the formation of radiolytic gases.

The reasons that one is inclined to speak of prompt and delayed coefficients of reactivity is that for power bursts of low reactivity and correspondingly long periods, the delayed effects may play a great part in the shut-down mechanism. However, if one supposes a burst with a very short period then it is obvious that these delayed effects will not have had time to act. What constitutes a "short time" can be answered by determining the heat transfer time constant of the fuel elements. This, in part, is the subject of the proposed investigation. Iriarte (27) reported heat transfer time constants for infinitely long cylindrical UO_2 fuel elements surrounded by a helium film which served as a thermal bond and which were clad with zirconium, stainless steel or aluminum. While the data presented by Iriarte are not applicable to the SPERT-I or TRIGA Reactors, the techniques may be useful in determining the relative importance of the delayed effects.

Before considering further the scope of this project it is informative to investigate the prior work in the field. During the early summer of 1954 a series of experiments were made on the BORAX-I Reactor to investigate the ability of the reactor, when operated in the subcooled (non-boiling)

condition, to protect itself against the results of sudden, artificially induced increases in reactivity. Inasmuch as this set of experiments completed the program for the BORAX-I Reactor, the final runaway experiment was intentionally made under conditions which led to destruction of the reactor. In the final experiment, a control rod worth four per cent $\underline{k_{eff}}$ was ejected from the reactor core, inducing an exponential power increase which had a period of 2.6 milliseconds. This final experiment resulted in a melting of most of the fuel plates and failure of the reactor tank. Fuel plate fragments were scattered for a distance of 200 to 300 feet (8). This set of experiments, along with the earlier operation of the BORAX-I, (7), established two important safety axioms for water moderated reactors. First, for any given system there is a reactivity insertion beyond which the reactor cannot react fast enough to shut itself down before damage is done, and second, water moderated reactors can be designed to have a high degree of inherent self-protection against the effects of sudden large reactivity increases. A less critical fact that resulted from these tests was that if the transients are started at boiling conditions (such as in a boiling water reactor), the maximum power and fuel-plate temperature reached are less than if the transient is started with the reactor in a subcooled condition. This is as would be expected if void formation due to boiling were the shut-down mechanism, since for the subcooled system the reactor could actually achieve a stable positive period before any negative reactivity would result. This, along with the observation of large quantities of steam and water being expelled from the system, led to the conclusion that it was the void formation which was shutting the reactor down. With this background in mind, the Atomic Energy Commission set out on an intensified program

to determine empirically the safe upper operating limits on each class of reactors including the pressurized and unpressurized, boiling and non-boiling thermal reactors, both heterogeneous and homogeneous, and fast-reactor systems (23). This program initiated the SPERT (Special Power Excursion Reactor Tests) and KEWB (Kinetic Experiments on Water Boilers) programs. SPERT is the heterogeneous reactor test facility and KEWB is the design for the same type of tests on a homogeneous reactor test facility. In 1956, W. B. Nyer, et. al. (39), reported the results of the initial transient test on the SPERT-I facility and concluded that the SPERT-I (43) reactor demonstrated qualitatively the results of the BORAX-I experiments although SPERT-I was more stable after the initial power burst. Factors which could contribute to this difference in behavior were the known differences in the fuel assembly construction, the possible differences in the effective void coefficients, and/or the differences in the reflector-tank environment. Another important point discovered at this time was that there was a unique relationship between the peak power and the transient period. The data were fitted rather well by two straight lines on a plot of log reciprocal period versus peak power. The slopes of these lines were approximately 0.8 and 1.7 for the lower and upper regions respectively. The point of intersection occurred at $\underline{\Delta k}$ = 0.74%, or about prompt critical. In June of 1957, R. W. Miller reported on some interesting work in analyzing the reactivity behavior during SPERT-I transients (34). Miller pointed out that it was not necessary to remove all of the initially inserted reactivity to limit a power excursion. This is a result of the delayed neutrons, which for a very short period excursion, do not contribute to the flux during the rise in power. Thus, for short excursions the reactivity

compensation at maximum power need only be $\Delta k(1-\beta)-\beta$. Δk is the initial change in k_{eff} and β is the delayed neutron fraction. The total Δk must be accounted for in very long period transients. In the case of intermediate period transients the compensated reactivity was calculated as a function of the reciprocal period by numerical solution of the reactor kinetics equations using the experimentally determined power traces. Later in 1957, G. O. Bright, et. al., suggested a model for reactor burst behavior (2), based on the earlier work of Klaus Fuchs at Los Alamos (21). This model postulated a shutdown effect proportional to the energy release. However, the model provided no method for the shutdown energy to be removed and allowed for no time-delays between the energy release and the appearance of the shutdown effect. Much later this model was further modified by S. G. Forbes (13) who let the shutdown effect be proportional to the energy release raised to some power, n, and allowed for some arbitrary delay time. In this analysis the model was tested against some experimental transients from SPERT-I and values of n from 1.5 to 2 were successively used. The results also showed that the delay time was significant in matching the data; however no exact value of the delay time was determined. The overall effect of these works is to convince one that the primary shutdown mechanisms are intimately tied up with the energy release although no real information on the exact phenomena can be found. In 1958, Griffing and Deverall (10) coupled the energy shut-down model with the reactor kinetics equation including six delay groups and again showed that the energy model could describe qualitatively the power traces obtained in the SPERT-I transients even long after the initial burst. This work used a mathematical structure of the shut-down equation considered much earlier (1951)

by Chernick (4) for no delay groups and in 1956 by Margulies (31) with one delay group.

In December of 1958, Deverall and Griffing (9) reported the first attempts at trying to relate the shutdown reactivity with the thermodynamic characteristics of the SPERT-I system. In this report the change in reactivity for transients with long periods was related to the temperature rise in the moderator alone although the fact was recognized that the fuel element temperature rise should also be considered. Considering only the moderator temperature rise, they found reactivity compensations at peak power that were roughly one-half of those reported by Miller (34). Since they felt that the data with which they were working were only accurate to within a factor of two, they did not pursue the investigation further. In January of 1958, Horning of Ramo-Woolridge reported on a model for transients in SPERT-I (18). This report develops a general model, taking into account the void formation and the thermal expansion. However, no real attempt was made to interpret these constants in terms of the distribution of energy in the fuel and moderator or the nuclear and thermodynamic constants of the system. Another report, bound under the same cover, by H. C. Corben (26) treats the problem of oscillations found after the burst as the power approaches some steady state level. Although the mechanism responsible for the oscillations need not be the same as the shut-down mechanism, there is certainly the possibility that they are one and the same. A third report by G. Birkhoff (26) treats the problem of void formation from the point of view of the growth of bubbles. The "Bubble Void" is probably the dominant shutdown effect in a certain type of excursion such as in the initial BORAX-I experiments. The analytic representation of this effect is the least known and thus is being widely

sought. An extensive study of bubble formation including a critical review of the literature, an evaluation of the merits of purely theoretical approaches to the development of a void model, and an investigation of the possible formulation of the nucleate boiling void was reported by the Vitro Engineering Company in May of 1959 (28).

Although an exact definition of bubble formation may not be within the scope of this investigation, the determination of the heat flow into the moderator and the transient temperature distribution in the moderator should shed some light on even this difficult problem.

In July of 1958, J. C. Haire (25) reported the results of a great number of the SPERT-I transients. This report presented data on the reactor power, fuel plate surface temperatures and pressures as a function of time during the transients. These are the data that will be used extensively in the initial phases of the investigation proposed herein.

During late 1958 and 1959, several models were proposed to investigate and explain the inherent shutdown characteristics of the SPERT-I reactor. The "Conduction Boiling" model suggested by S. G. Forbes (14) is certainly credible in that it takes into account the flow of heat into the moderator in a much more exact manner than any of the earlier investigations. This model was quite successful in predicting the power, energy release and temperature at the time of peak power as a function of the reciprocal period. However the model still represents the shutdown mechanism in terms of empirical parameters. Also the non-boiling shutdown effects are not taken into account. The "Clipped Exponential" model suggested by R. W. Miller (35) made some very useful assumptions on the shape of the reactor power burst to ease the analytical solution of kinetics equations. While this model produced some useful criteria in the understanding of self-shutdown

it again made use of lumped parameters which were not easily interpreted in terms of the thermodynamic and nuclear characteristics of the system. E. T. Clark (5,6) as early as 1956 had postulated a prompt fission product having a large absorption cross section for thermal neutrons as an explanation of the self-shutdown of power excursions observed in the SPERT-I Reactor. Later evidence (42) seems to indicate that this model is less likely to be valid than the more conventional models.

Also in 1958, General Atomic introduced their TRIGA Reactor with zirconium hydride moderator which demonstrated a larger prompt shutdown mechanism than either the BORAX-I or the SPERT-I (41). In this case the reactor was designed to have a shutdown mechanism which would act by hardening the thermal neutron energy spectrum thus increasing the thermal leakage to cause shutdown. The reason for choosing this effect was that they felt that it would act more quickly and thus provide a safer reactor than the accepted moderator expansion and expulsion mechanisms. The spectrum effect, so important in the TRIGA Reactors must also act to some extent in SPERT-I. The amount of this effect has apparently never been determined.

P. French, in 1959, (18) reported an attempted solution of the transient heat conduction equation in the fuel and moderator to determine the temperature distribution. However, this work appears to be in error in that he forced a separation of variables solution on the equations whereas the spatial and time dependence cannot be expressed as a simple product except after sufficiently long times so that the transient term has disappeared. H. L. McMurry (32) reported the temperature distribution in a fuel plate, cladding and moderator with exponentially rising power for pure conduction. This is an excellent piece of work but the mathematical

model turns out to be more difficult than is either warranted or necessary for the analysis of the SPERT-I data. The McMurry report makes mention of earlier work by H. Greenspan (24) on the same problem with similar results. Several investigators, Kattwinkel (29), Kirchenmayer (30), Stein (40), Epel (11), Arpaci and Clark (1), Ermakov and Ivanov (12), report analytical solutions to the transient heat conduction equation. However, none of them were working on the problem with reference to the SPERT-I investigations and their results are not directly applicable to the use of the SPERT-I experimental measurements. As a result the temperature distribution in the fuel and moderator during a transient is not well known to date.

In July of 1959, Forbes, et. al. (16), summarized the work done up to that time. They showed that by fitting empirically the "Conduction Boiling" model to the SPERT-I data and including the effect of moderator and fuel element expansion they could fit the experimental compensated reactivity at peak power versus reciprocal period curves for values of the reciprocal period greater than 5 seconds^{-1}. They also showed that for values of the reciprocal period greater than 20 seconds^{-1}, the steam void contribution to reactivity was considerable. For the longer period region they postulated an additional shutdown effect from radiolytic gases.

This, of course, lends considerable credence to the "Boiling Conduction" model, however it does have one glaring shortcoming. To extend it to another reactor requires at least one transient burst experiment to determine the parameters. However, if the fundamental mechanism were understood exclusively in terms of the basic nuclear, thermodynamic and hydrodynamic characteristics of the system, one could confidently predict the limits of safe operations for different reactor systems.

The satisfactory application of the "Boiling Conduction" model led to two sets of experiments designed to test that model and the postulated radiolytic gas effect. The first set of experiments (20) showed with reasonable certainty that radiolytic gas formation was not a primary contributor to self-shutdown in the SPERT-I Reactor. The second set of experiments (19) consisted of coating all of the fuel plates with approximately five mils of insulation, Lithocote LC-34, and of performing transient tests on the reactor. Some of the tests were run with transients of such magnitude that boiling would occur in the bare core and not in the insulated core. The remaining tests involved transients in which there would be boiling in both cores, and the differences in heat transfer rates were expected to be reflected as changes in the reactor behavior. Power burst shapes for transients of the same period in both the bare and insulated cores were essentially the same. In view of the identical reactor behavior for the bare and insulated tests, it would appear that the core insulation produced no appreciable effects on the shutdown mechanism. Since it seems reasonable to assume that any shutdown effect due to boiling would be effected by the core insulation, boiling would not seem to play any part in the self-shutdown mechanism. However, if the heat transfer rate was small enough there would be negligible temperature drop across the insulation and thus the effect on boiling might be unchanged by the insulation. This experiment shows clearly the need for detailed calculation of the temperature distributions in the fuel and moderator during the transients. One final report should be cited in this summary. In April of 1960, Miller (36) reported on some photographic investigations of boiling during transients

in SPERT-I. These results clearly indicated that boiling was an important agent in the initial reactor self-shutdown whenever the fuel plate temperature was sufficiently high.

2.0 THEORY

2.1 Derivation of Equations

The direct approach to determining the self-shutdown effects must include the determination of the temperature distributions in the fuel elements and moderator throughout the core during a transient. This problem can be accomplished by investigating the exact temperature distribution in a center element only and relating all effects to this center element. The exact solution of the multi-region, transient heat conduction equations even for a single fuel element, making use of only the power versus time data from the SPERT-I transients, is exceedingly difficult as pointed out in the work by McMurry (32). However, if use is made of the available fuel plate surface temperature data as well as the power data during a transient, the problem is reduced to two single region problems. Although the solutions are much simplified over the two region problem, they are still complicated and therefore have been programmed for the Kansas State University IBM 650 computer.

The methods for determining the steady state temperature distribution throughout a unit cell of thermal, heterogeneous liquid-moderated reactor are discussed in Nuclear Engineering (41) by C. F. Bonilla and in Nuclear Reactor Physics (37) by R. L. Murray. It will be considered sufficient for this work to outline the differences that must be accounted for in transient operation.

The partial differential equation for conductive heat transfer applicable in the fuel and moderator of a nuclear power reactor during

a transient but before boiling is established is

$$\nabla^2\theta(x,t) + \frac{q(x,t)}{k} = 1/\alpha \frac{\partial\theta(x,t)}{\partial t} \tag{1}$$

where ∇^2 is the Laplacian operator (44), $\underline{\theta}$ is the temperature rise above the initial temperature $[T(x,t)-T_o]$, $\underline{x}$ is the position variable, $\underline{k}$ is the mean thermal conductivity, $\underline{\alpha}$ is the mean thermal diffusivity, $\underline{t}$ is the time variable and $\underline{q}$ is the volumetric heat generation rate. This equation in slab and cylindrical geometries is directly applicable to the analysis of transient behavior in a reactor following a change in reactivity. The derivation of solutions to this equation in the fuel and moderator during power transients will be shown in detail for slab geometry (Appendix A) and the important elements of the solution in cylindrical geometry will be tabulated in Section 2.2.1.

In any nuclear reactor there are heating effects due directly to fission fragments and to the attenuation of other nuclear particles. Within the fuel element, fission heating far overrides the other attenuation effects. Therefore, the heat generation rate, $\underline{q_f(x,t)}$, in the fuel elements is proportional to the thermal neutron flux. The neutron flux during a transient can be expressed as a simple product of the spatial and the time dependencies. Thus the heat generation rate is also separable in space and time as shown in equation (2).

$$q_f(x,t) = f_f(x) \quad g_f(t) \tag{2}$$

The spatial dependence, $\underline{f_f(x)}$, can be obtained easily from the steady state analysis (40) and is given in equation (3).

$$f_f(x) = q_{oo} \cosh \kappa x \tag{3}$$

Here q_{oo} is the heat generation rate at the center of the fuel, κ is the inverse thermal neutron diffusion length and x is the distance from the center of the fuel.

The time dependence, $g_f(t)$, of the neutron flux and thus the heat generation rate is expressed as the sum of exponentials, thus

$$g_f(t) = \sum_{j=1}^{s} a_j e^{\lambda_j t} \tag{4}$$

Substituting equations (3) and (4) into equation (2), an expression for $q_f(x,t)$ is obtained; that is

$$q_f(x,t) = \sum_{j=1}^{s} q_{oo} \cosh(\kappa x)\, a_j e^{\lambda_j t} \tag{5}$$

Substituting equation (5) into equation (1) yields the differential equations which must be solved to obtain the temperature distribution in the fuel, that is

$$\nabla^2 \theta_f(x,t) + \sum_{j=1}^{s} \frac{q_{oo} \cosh(\kappa x)\, a_j e^{\lambda_j t}}{k} = 1/\alpha \frac{\partial \theta_f(x,t)}{\partial t} \tag{6}$$

Now from investigation of the heat generation rate, $q_m(k,t)$ in the moderator, it is noted that there is no heating due to fission, but there is heating due to nuclear particles which stream out of the fuel and are attenuated in the moderator. The moderator heating is approximately 5 to 7% (3) of the recoverable energy from fission and is sufficiently uniform in space to be so considered. Therefore, $q_m(x,t)$ in the moderator is independent of the spatial variable but still is time dependent as shown in quation (7).

$$q_m(x,t) = \sum_{j=1}^{s} F\, a_j e^{\lambda_j t} \tag{7}$$

The heating effects in the moderator are proportional to the neutron flux in the fuel; thus F is the fraction of the recoverable energy released in the fuel which is dissipated in the moderator.

Substitution of equation (7) into equation (1) yields the differential equation to be solved for the temperature distribution in the moderator, that is

$$\nabla^2\theta_m(x,t) + \sum_{j=1}^{s} \frac{F\, a_j e^{\lambda_j t}}{k} = 1/\alpha \, \frac{\partial\, \theta_m(x,t)}{\partial t} \qquad (8)$$

Equations (6) and (8) form the general set which must be solved to obtain the temperature distribution in a unit cell of the reactor during a transient. The geometry of the unit cell in which these equations must be applied is shown in Figure 1. The simultaneous solution of these equations is extremely difficult. The availability of the fuel element surface temperature as a function of time during the transients greatly simplifies this situation. The problem is reduced to solving the equations independently in the fuel and moderator, using the experimentally measured temperatures at the interface as a boundary condition for both equations. The other boundary condition necessary in each case is a zero heat flow condition at the center of the unit cell for the fuel regions and at the boundary of the unit cell for the moderator.

2.2 Analytical Solutions

The time dependent thermal diffusion equations in the fuel and moderator can be solved for the temperature distribution assuming that conduction is the primary mode of heat transfer. The equation will hold for all time in the fuel plate. In an attempt to represent as well as

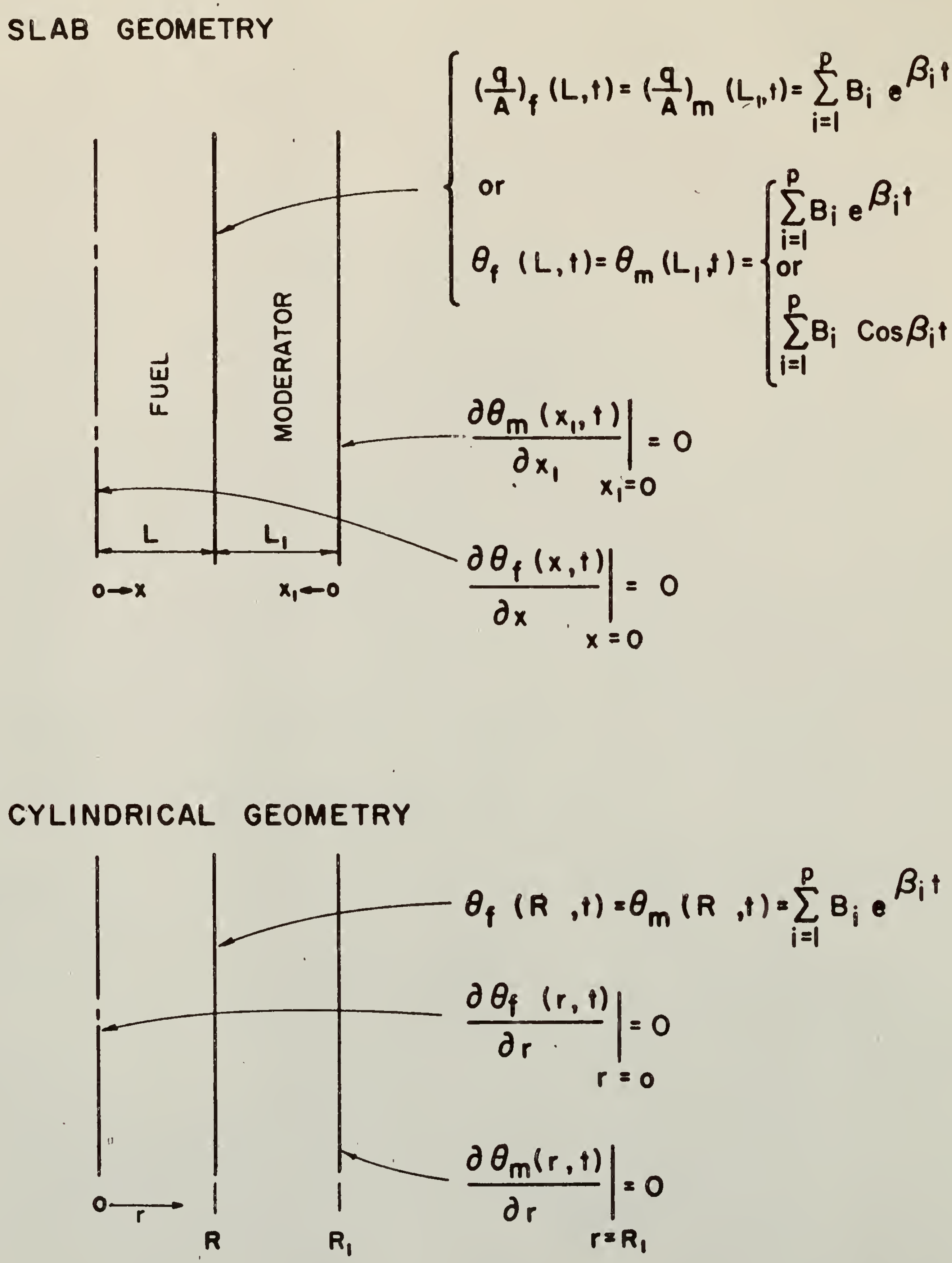

Figure 1. Geometry and boundary conditions in the unit cell used to determine the temperature distributions.

possible the experimental data which is used as input to this problem and to allow some flexibility in the application of these equations, the differential equations were solved subject to several forms of representation of the boundary conditions and the forcing functions. In one case the problem was solved in cylindrical geometry.

In slab geometry the one-dimensional solutions to the transient heat transfer equation, in which the time dependence of heat generation rate and the fuel element surface temperature are represented empirically as $\sum_{j=1}^{s} A_j e^{\lambda_j t}$ and $\sum_{i=1}^{p} B_i e^{\beta_i t}$ respectively, are derived in Appendix A and are given here as

$$\theta_f(x,t) = \sum_{i=1}^{p} \frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\, x\right) e^{\beta_i t}}{\cosh \sqrt{\frac{\beta_i}{\alpha}}\, L} - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\left(\frac{n\pi x}{2L}\right) e^{\frac{n^2\pi^2\alpha}{4L^2} t}}{(L^2/n\pi\alpha)\sin\frac{n\pi}{2}}$$

$$\times \left\{ \sum_{i=1}^{p} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} + \sum_{j=1}^{s} \frac{q_{oo}\,\alpha\, A_j \cosh(\kappa L)}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)} \right\} \quad (9)$$

$$- \sum_{j=1}^{s} \frac{A_j\, q_{oo}\, \alpha e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{ \cosh \kappa x - \frac{\cosh(\kappa L)\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, x\right)}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)} \right\}$$

and

$$\theta_m(x,t) = \sum_{i=1}^{p} \frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\, x_1\right) e^{\beta_i t}}{\cosh \sqrt{\frac{\beta_i}{\alpha}}\, L_1} - \sum_{n=1,3,5,\ldots}^{\infty} \frac{\cos \frac{n\pi x_1}{L} e^{\frac{-n^2\pi^2\alpha}{4L^2} t}}{(L_1^2/n\pi\alpha)\sin\left(\frac{n\pi}{2}\right)}$$

$$\times \left\{ \sum_{i=1}^{p} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L_1^2} + \beta_i} + \sum_{j=1}^{s} \frac{\alpha F A_j}{k\left(\frac{n^2\pi^2\alpha}{4L_1^2}\right)\left(\frac{n^2\pi^2\alpha}{4L_1^2} + \lambda_j\right)} \right\} \quad (10)$$

$$+\sum_{j=1}^{s} \frac{\alpha F A_j e^{\lambda_j t}}{k \lambda_j} \left\{ 1 - \frac{\cosh \sqrt{\frac{\lambda_j}{\alpha}}\, x_1}{\cosh \sqrt{\frac{\lambda_j}{\alpha}}\, L_1} \right\}$$

in the fuel and moderator, respectively.

The equivalent solutions in cylindrical geometry are derived in Appendix A and are given here as

$$\theta_f(x,t) = \sum_{i=1}^{p} \frac{B_i I_o \left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right) e^{\beta_i t}}{I_o \sqrt{\frac{\beta_i}{\alpha}}\, R} + \sum_{n=1}^{\infty} \frac{J_o\left(\frac{\omega_n r}{R}\right) e^{-\frac{\omega_n^2 \alpha}{R^2} t}}{\frac{R^2}{2\omega_n \alpha} J_1(\omega_n)}$$

$$\times \left\{ \sum_{i=1}^{p} \frac{B_i}{\frac{\omega_n^2 \alpha}{R^2} + \beta_i} + \sum_{j=1}^{s} \frac{q_{oo}\, \alpha A_j I_o(\kappa R)}{k \left(\frac{\omega_n^2 \alpha}{R^2} + \lambda_j\right)\left(\frac{\omega_n^2 \alpha}{R^2} + \alpha\kappa^2\right)} \right\} \tag{11}$$

$$- \sum_{j=1}^{s} \frac{A_j q_{oo}\, \alpha e^{\lambda_j t}}{k (\alpha\kappa^2 - \lambda_j)} \left\{ I_o(\kappa r) - \frac{I_o(\kappa R)\, I_o\left(\sqrt{\frac{\lambda_j}{\alpha}}\, r\right)}{I_o\left(\sqrt{\frac{\lambda_j}{\alpha}}\, R\right)} \right\}$$

and

$$\theta_m(x,t) = \sum_{i=1}^{p} B_i \left\{ \frac{K_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right) + I_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) K_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right)}{K_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, R\right) + I_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) K_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, R\right)} \right\} e^{\beta_i t}$$

$$+ \sum_{n=1}^{\infty} \frac{2\sqrt{\rho_n \alpha}\left[K_1\left(\sqrt{\frac{\rho_n}{\alpha}}\, R\right) I_o\left(\sqrt{\frac{\rho_n}{\alpha}}\, r\right) + I_1\left(\sqrt{\frac{\rho_n}{\alpha}}\, R\right) K_o\left(\sqrt{\frac{\rho_n}{\alpha}}\, r\right) \right] e^{\rho_n t}}{R\left[K_1\left(\sqrt{\frac{\rho_n}{\alpha}}\, R_1\right) I_1\left(\sqrt{\frac{\rho_n}{\alpha}}\, R\right) - K_1\left(\sqrt{\frac{\rho_n}{\alpha}}\, R\right) I_1\left(\sqrt{\frac{\rho_n}{\alpha}}\, R_1\right) - I_o\left(\sqrt{\frac{\rho_n}{\alpha}}\, R_1 + I_o\left(\sqrt{\frac{\rho_n}{\alpha}}\, R\right) K_o \sqrt{\frac{\rho_n}{\alpha}} R\right) \right]}$$

$$\times \sum_{i=1}^{p} \left\{ \frac{B_i}{\rho_n - \beta_i} - \sum_{j=1}^{s} \frac{\alpha F A_j}{k\, \rho_n(\rho_n - \lambda_j)} \right\} \tag{12}$$

$$+ \sum_{j=1}^{s} \frac{\alpha F A_j e^{\lambda_j t}}{k} \left\{ \frac{K_1\sqrt{\frac{\lambda_j}{\alpha}} R_1 I_o \sqrt{\frac{\lambda_j}{\alpha}} r + I_1\sqrt{\frac{\lambda_j}{\alpha}} R_1 K_o \sqrt{\frac{\lambda_j}{\alpha}} r}{K_1\sqrt{\frac{\lambda_j}{\alpha}} R_1)\, I_o \sqrt{\frac{\lambda_j}{\alpha}} R) + I_1(\sqrt{\frac{\lambda_j}{\alpha}} R_1) K_o (\sqrt{\frac{\lambda_j}{\alpha}} R)} \right\}$$

where ω_n's are the roots of the equation, $J_o(x) = 0$

and ρ_n's are the roots of the equation,

$$[K_1(\sqrt{\tfrac{S}{\alpha}} R_1)\, I_o(\sqrt{\tfrac{S}{\alpha}} R) + I_1(\sqrt{\tfrac{S}{\alpha}} R_1)\, K_o(\sqrt{\tfrac{S}{\alpha}} R)] = 0$$

The solutions to the transient heat transfer equations in slab geometry in which the time dependence of the heat generation rate and the first derivative of the surface temperature with respect to x are $\sum_{j=1}^{s'} A_j e^{\lambda_j t}$ and $\sum_{i=1}^{p'} B_i e^{\beta_i t}$ respectively are derived in Appendix A and are given here as

$$\theta_f(x,t) = \sum_{j=1}^{p'} \frac{B_i \cosh(\sqrt{\frac{\beta_i}{\alpha}} x)\, e^{\beta_i t}}{\sqrt{\frac{\beta_i}{\alpha}} \sinh(\sqrt{\frac{\beta_i}{\alpha}} L)} - \frac{B_i \alpha}{B_i L} - \sum_{n=1}^{\infty} \frac{\cos \frac{n\pi x}{2L}\, e^{\frac{n^2\pi^2\alpha}{4L^2} t}}{(L/\alpha) \cos n\pi}$$

$$\times \left\{ \sum_{i=1}^{p'} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} + \sum_{j=1}^{s'} \frac{q_{oo}\, \alpha A_j\, \kappa \sinh \kappa L}{k(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j)(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2)} \right\} \tag{13}$$

$$-\sum_{j=1}^{s'} \frac{q_{oo}\, \alpha A_j e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{ \cosh \kappa x - \frac{\kappa \sinh(\kappa L) \cosh(\sqrt{\frac{\lambda_j}{\alpha}} x)}{\sqrt{\frac{\lambda_j}{\alpha}} \sinh(\sqrt{\frac{\lambda_j}{\alpha}} L)} \right\}$$

$$- \frac{q_{oo}\, \alpha A_j \sinh \kappa L}{k\, \lambda_j L\kappa}$$

and

$$\theta_m(s,t) = \sum_{i=1}^{p'} \frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\, x\right) e^{\beta_i t}}{\sqrt{\frac{\beta_i}{\alpha}} \sinh \sqrt{\frac{\beta_i}{\alpha}}\, L} - \frac{B_i \alpha}{\beta_i L}$$

$$- \sum_{n=1}^{\infty} \frac{\cos \frac{n\pi x}{2L}\, e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{(L/\alpha) \cos n\pi} \left\{ \sum_{i=1}^{p'} \frac{B_i \alpha}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} \right\} \qquad (14)$$

$$+ \sum_{j=1}^{s'} \frac{\alpha F A_i}{k(\lambda_j)} \left(e^{\lambda_j t} - 1\right)$$

in the fuel and moderator, respectively.

The solution to the transient heat transfer equation in slab geometry in which the time dependence of the heat generation rate and the surface temperature are represented by $\sum_{j=1}^{s} A_j e^{\lambda_j t}$ and $\sum_{i=1}^{p} B_i \cos \beta_i t$ respectively are derived in Appendix A and are given here as

$$\theta_f(x,t) = \sum_{i=1}^{p} B_i Z_i^{\frac{1}{2}} \cos(\beta_i t + \varphi_i) - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos \frac{n\pi x}{2L}\, e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{(L^2/n\pi\alpha) \sin \frac{n\pi}{2}}$$

$$\times \left\{ \sum_{i=1}^{p} \frac{B_i \left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\left(\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2\right)} + \sum_{j=1}^{s} \frac{q_{oo}\, \alpha A_j \cosh \kappa L}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)} \right\} \qquad (15)$$

$$-\sum_{j=1}^{s} \frac{q_{\infty}\,\alpha\, A_j\, e^{\lambda_j t}}{k\,(\alpha\kappa^2 - \lambda_j)} \left\{ \cosh \kappa x - \frac{\cosh(\kappa L)\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, x\right)}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)} \right\}$$

and

$$\theta_m(x,t) = \sum_{i=1}^{p} B_i Z_i^{\frac{1}{2}} \cos(\beta_i t + \varphi_i) + \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos \frac{n\pi x}{2L}\, e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{(L^2/n\pi\alpha)\sin \frac{n\pi}{2}}$$

$$\times \left\{ \sum_{i=1}^{p} \frac{B_i \frac{n^2\pi^2\alpha}{4L^2}}{\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2} - \sum_{j=1}^{s} \frac{\alpha F A_j}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)} \right\} \qquad (16)$$

$$+\sum_{j=1}^{s} \frac{\alpha F A_j\, e^{\lambda_j t}}{k\,\lambda_j} \left\{ 1 - \frac{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, x\right)}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)} \right\}$$

where

$$Z_i = \frac{\cos^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, x\right)\cosh^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, x\right) + \sin^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, x\right)\sinh^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, x\right)}{\cos^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right)\cosh^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right) + \sin^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right)\sinh^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right)}$$

and

$$\varphi_i = \tan^{-1} \frac{\sin\left(\sqrt{\frac{\beta_i}{2\alpha}}\, x\right)\sinh\left(\sqrt{\frac{\beta_i}{2\alpha}}\, x\right)}{\cos\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right)\cosh\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right)} - \tan^{-1} \frac{\sin\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right)\sinh\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right)}{\cos\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right)\cosh\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right)}$$

2.2.1 Temperature Distributions. Equations (15) and (16) of the previous section can be evaluated to obtain the temperature as a function of position and time in any unit cell of a reactor if the heat generation rate, $q(x,t)$, and the fuel surface temperature, $\theta_f(L,t)$, are known and can be expressed in the appropriate analytical form. The experimental values of these variables during applicable transient tests on the SPERT-I reactor system were obtained in graphical form from "Sub-cooled Transient Tests in the SPERT-I-A Reactor - Experimental Data" by J. C. Haire (25). Numberical power and temperature data were obtained from the graphs. These numerical data were normalized to a zero initial temperature then fit empirically by an even trigonometric series, $\sum_{i=1}^{p} A_i \cos \beta_i t$, for the temperature traces. The power traces were reduced to give the heat generation rate in the center of a central fuel element and moderator region and then fit empirically by an exponential series, $\sum_{j=1}^{s} A_j e^{\lambda_j t}$.

The reduction of the power data to give the appropriate heat generation rate in the fuel region was accomplished in the following manner. The SPERT-I core contained 28 assemblies, 51 plates per assembly and an active volume of 7.523 cm^3 per plate. Therefore,

$$\overline{H}_{plate}(t) = \left(\frac{P(t) \times 10^6}{28 \text{ assemblies}}\right) \left(\frac{1 \text{ assembly}}{51 \text{ plates}}\right) \left(\frac{1 \text{ plate}}{7.523 \text{ cm}^3}\right) \qquad (16)$$

$$= 97.78\ P(t) \text{ watts/cm}^3$$

was the average heat generation rate in an average fuel plate, where $P(t)$ is the total power in megawatts. Converting this to the required dimensions of cal/sec-cm^3 yielded $\overline{H}_{plate}(t) = 23.37\ P(t)$ cal/sec cm^3. The heat generation rate in the center of the fuel plate is found in terms of the average heat generation rate since the heat generation rate is proportional to the neutron flux distribution.

$$\bar{H}_{plate}(t) \doteq \frac{\int_o^L C\,\Phi\,(x)\;Adx}{\int_o^L Adx} = \frac{\int_o^L H_o \cosh \kappa x\;dx}{L} = \frac{H_o \sinh \kappa L}{\kappa\;L} \qquad (17)$$

where L is half-thickness of plate, 0.0254 cm,

and $\bar{H}_o$ is heat generation rate at center of the average fuel plate.

κ was determined from the neutron transport theory relationship for heavy absorbers (44),

$$\frac{\kappa}{\Sigma_{tot}} = \tanh \frac{\kappa}{\Sigma_s} \qquad (18)$$

to be 0.7973 cm^{-1} .

$$\kappa L/\sinh \kappa L \doteq \frac{0.02025}{0.02025} = 1.0 \qquad (19)$$

Therefore,

$$\bar{H}_{of}\;(t) = \bar{H}_{plate}(t) \doteq 23.37\;P(t)\;cal/sec\;cm^3. \qquad (20)$$

The correction from the average fuel element to the one of interest, a central fuel element, required a maximum to average correction. Therefore,

$$H'_{of}(t) = \bar{H}_{of}(t)\;(\Phi_{max}/\bar{\Phi}) = 23.37\;P(t)\;(1.9) = 44.38\;P(t)\;cal/cm^3sec. \qquad (21)$$

The final correction was to assume that approximately 5% of the power was generated in the moderator. Therefore,

$$H_{of}(t) = 0.95\;H'_{of}(t) = 42.16\;P(t)\;cal/cm^3sec. \qquad (22)$$

The relation used to calculate the heat generation rate in the moderator was

$$q_m(t) = 0.05\;\bar{q}_f(t)\;\frac{V_f}{V_m} = 0.05\;\bar{q}_f(t)\;\frac{L_f}{L_m} = 0.05\;H_{of}(t)\;(\frac{0.0254}{0.071755}) = 0.017695\;H_{of}(t) \qquad (23)$$

where $\bar{q}_f$ is average heating rate in the fuel,

V_f is volume of the fuel,

V_m is volume of the moderator,

L_f is half-thickness of the fuel,

L_m is half-thickness of the moderator,

and q_m is the heat generation rate in the moderator.

The above equation assumes that there is a flat spatial distribution of the heating rate, that approximately 5% of the total heat generation takes place in the moderator and that the average heat generation rate in the fuel is well approximated by the heat generation rate in the center of the fuel.

As previously mentioned that data from the temperature traces were fit with a finite number of terms of a Fourier series of the form

$$\sum_{n=0}^{p} b_n \cos \frac{2\pi nt}{a} \tag{24}$$

where

$$b_o = \frac{2}{a} \int_o^a y\,(t)\,dt,$$

$$b_n = 1\,a \int_o^a y(t) \cos \frac{2\pi nt}{a}\,dt,$$

$y(t)$ = experimental temperature trace data,

and a = interval of periodicity.

The data reduced from the power traces, actually $H_{of}(t)$, were fitted with a finite number of terms of an exponential function of the form,

$$\sum_{j=1}^{s} A_j\, e^{\lambda_j t}, \tag{25}$$

where $\underline{A_j}$ and $\underline{\lambda_j}$ were parameters which were determined by trial and error to give the best fit. The best fit parameters were determined by means

of an IBM-650 computer program described in Appendix B. This program resulted from a very minor modification of one written by L. R. Foulke (17). The data for $H_{of}(t)$ and the approximating equations are shown in Figures 8 through 11.

The thermal, nuclear and geometric constants used in determining the temperature distributions are given in Table 1.

Table 1. Constants Used to Evaluate the Temperature Distributions

Constant	Fuel	Moderator
α, Thermal Diffusivity, cm^2/sec	0.82	0.001512
κ, Inverse Diffusion Length, cm^{-1}	0.7973	0.0
L, Half-thickness of Region, cm	0.0254	0.071755
k, Thermal Conductivity, cal/cm sec ^{o}C	0.5002	0.001488

<u>2.2.2 Surface Heat Flow</u>. The heat flow rate out of the fuel and into the moderator as a function of time was evaluated by forming the partial derivative with respect to position evaluating it at the outside of the respective region and multiplying by the respective thermal conductivity. The heat flow out of the fuel and into the moderator are, respectively,

$$(q/A)_f(t) = -k_f \left. \frac{\partial \theta_f(x,t)}{\partial x} \right|_{x=L} \tag{26}$$

and

$$(q/A)_m(t) = -k_m \left. \frac{\partial \theta_m(x,t)}{\partial x} \right|_{x=L} , \tag{27}$$

Evaluating the above equations yields in the fuel

$$(q/A)_f(t) = -k_f \Bigg\{ \sum_{i=1}^{p} \left(\sqrt{\frac{\beta_i}{2\alpha}}\right) B_i \;(D_i \cos \beta_i t + E_i \sin \beta_i t)$$

$$+ \sum_{n=1,3,5,\cdots}^{\infty} \frac{\left(\frac{n\pi}{2L}\right) e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{(L^2/n\pi\alpha)} \left[\sum_{i=1}^{p} \frac{B_i \left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2} + \sum_{j=1}^{s} \frac{q_{\infty}\, \alpha\, A_j \cosh \kappa L}{k_f \left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)} \right]$$

$$+ \sum_{j=1}^{s} \frac{q_{\infty} \alpha\, A_j\, e^{\lambda_j t}}{k_f\; (\alpha\kappa^2 - \lambda_j)} \left[\frac{\sqrt{\frac{\lambda_j}{\alpha}} \cosh(\kappa L) \sinh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)} - \kappa \sinh \kappa L \right] \Bigg\}$$

and in the moderator

$$(q/A)_m(t) = -k_m \Bigg\{ \sum_{i=1}^{p} \left(\sqrt{\frac{\beta_i}{2\alpha}}\right) B_i \;(D_i \cos \beta_i t + E_i \sin \beta_i t)$$

$$+ \sum_{n=1,3,5,\cdots}^{\infty} \frac{\left(\frac{n\pi}{2L}\right) e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{L^2/n\pi\alpha} \left(\sum_{j=1}^{s} \frac{\alpha\, F\, A_j}{k_m \left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right) \left(\frac{n^2\pi^2\alpha}{4L^2}\right)} + \sum_{i=1}^{p} \frac{B_i \left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2} \right)$$

$$- \sum_{j=1}^{s} \frac{\alpha\, F\, A_j}{k_m\, \lambda_j} \left(\frac{\sqrt{\frac{\lambda_j}{\alpha}} \sinh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)} \right) \Bigg\},$$

$$\text{where } D_i = \frac{\cosh\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right) \sinh\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right) - \cos\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right) \sin\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right)}{\cos^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right) \cosh^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right) + \sin^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right) \sinh^2\left(\sqrt{\frac{\beta_i}{2\alpha}}\, L\right)}$$

$$\text{and } E_1 = \frac{\cosh(\sqrt{\frac{\beta_1}{2\alpha}}L)\sinh(\sqrt{\frac{\beta_1}{2\alpha}}L) + \cos(\sqrt{\frac{\beta_1}{2\alpha}}L)\sin(\sqrt{\frac{\beta_1}{2\alpha}}L)}{\cos^2(\sqrt{\frac{\beta_1}{2\alpha}}L)\cosh^2(\sqrt{\frac{\beta_1}{2\alpha}}L) + \sin^2(\sqrt{\frac{\beta_1}{2\alpha}}L)\sinh^2(\sqrt{\frac{\beta_1}{2\alpha}}L)} .$$

2.3 Reactivity Effects Due to Temperature Coefficient and Fuel Expansion

It is pointed out by Deverall and Griffing (9) that the temperature rise in the moderator in the central unit cell cannot be used directly to determine reactivity changes. "Since the temperature coefficient of reactivity, α, was determined under conditions of a uniform temperature throughout the core - a condition that does not exist in a transient - it is necessary to define a properly weighted average temperature. This average temperature would then produce the same change in reactivity as if an actual uniform temperature change of this amount had been made. This average is defined by

$$\overline{\Delta T} = \frac{\int I(\vec{x})\,\Delta T(\vec{x})\,d\vec{x}}{\int I(\vec{x})\,d\vec{x}} \tag{30}$$

where

$\Delta T(\vec{x})$ is the change of temperature at position $\vec{x}$,

$I(\vec{x})$ is the statistical importance at position $\vec{x}$,

and the integration is carried out over the whole volume of the reactor."

The authors also show that for SPERT I-A (17/28) core, the system under consideration,

$$\frac{\overline{\Delta T}}{\Delta T_{max}} = 0.65 \tag{31}$$

Therefore,

$$\Delta k = 0.65\,\alpha(T)\,\Delta T_{max} . \tag{32}$$

A value for α (t) of 0.9 x 10^{-4}($\Delta k/C^{o}$) was used yielding

$$\Delta k = (-5.85 \times 10^{-5} / C^{o}) \Delta T_{max}. \quad (33)$$

A similar problem was faced by Forbes (13). In determining the reactivity effect due to fuel plate expansion he stated,

"In order to obtain the reactivity change, the temperature distribution and void importance function in the core must be combined to obtain the dynamic reactivity coefficient as opposed to the static coefficient which applies only to uniform void distributions. Applying the observed distribution functions for temperature and void worth, it is found that the effective average temperature rise under dynamic conditions can be obtained from the temperature rise at the center of the core by the relation

$$\Delta\theta = 0.7 \, \Delta\theta_{max}. \quad (34)$$

The reactivity change due to plate expansion, Δk_1, will be

$$\Delta k_1 = \overline{\left(\frac{\partial k}{\partial v}\right)} \, 3 \, a \, v \, (0.7 \, \Delta\theta_{max}), \quad (35)$$

where $\overline{\left(\frac{\partial k}{\partial v}\right)}$ is the average void coefficient for the core,

a is the linear expansion coefficient of aluminum,

v is the volume of the aluminum which is heated (i.e., the volume of fuel plates proper),

$\overline{\Delta\theta}$" is the average temperature rise of the aluminum

and $\Delta\theta_{max}$ is the temperature rise at center of the core.

For the SPERT I-A (17/28) core the appropriate constants are the following:

$$a = 2.5 \times 10^{-5}/^{o}C$$

$$v = 2.8 \times 10^{4}\ cm^{3}$$

$$\overline{(\frac{\partial k}{\partial v})} = -3.5 \times 10^{-6}\ \Delta k/cm^{3}$$

Therefore, the expression for the reactivity change becomes

$$\Delta k_1 = (-5 \times 10^{-6} \frac{\Delta k}{{}^{o}C})\ (\Delta\theta_{max})." \qquad (36)$$

In order to avoid erroneously taking into account the void formation due to fuel element expansion in both the temperature coefficient and in the fuel element expansion calculation, the fuel element expansion was calculated only for the temperature rise in the fuel over the temperature rise in the moderator. Therefore, the reactivity effects due to the temperature coefficient, Δk_T, and due to the fuel plate expansion, Δk_E, are

$$\Delta k_T(t) = -5.85 \times 10^{-5}\ (\Delta k/{}^{o}C)\ \bar{\theta}_{mod}\ (t) \qquad (37)$$

and

$$\Delta k_E(t) = -5 \times 10^{-6}\ (\Delta k/{}^{o}C)\ (\bar{\theta}_{fuel}(t) - \bar{\theta}_{mod}\ (t)) \qquad (38)$$

where $\bar{\theta}_{mod}\ (t)$ is the average temperature rise in the moderator at time t

$\bar{\theta}_{fuel}(t)$ is the average temperature rise in the fuel at time t as obtained from Tables 2 through 5.

2.4 Reactivity Effects Due to Steam Formation

The calculation of the steam production was based on the same assumptions as those used in the "Conduction Boiling Model for Reactor Self-Shutdown" suggested by S. G. Forbes (15). In Forbes' work the steam volume, V_s, was assumed to be proportional to a fraction, f_{as}, of the energy, E_s, transferred to the moderator after the time boiling first occurred

in the core. The steam volume is given by

$$V_s = \frac{f_{as} E_s}{h_s} , \tag{39}$$

where h_s is the energy required to form a unit volume of steam from boiling water at standard pressure (1.35 watt-sec/cm^3 steam). The term f_{as} was regarded as a combination of factors involving the fraction of the energy actually forming steam during nucleate boiling (about 1%) and the fraction of the core heat transfer area, A, which is involved in boiling heat transfer (about 10%) (15). The reactivity effect of the steam is

$$\Delta k_s = V_s C_v = \frac{f_{as} E_s}{h_s} C_v , \tag{40}$$

where C_v is the void coefficient in the center of the core, ($C_v = -7.2 \times 10^{-6}$ $\Delta k/cm^3$ for Spert I-A). In this investigation the factor f_{as} was divided into its two components, the fraction of the core involved in boiling, f_a, and the fraction of the energy actually forming steam during nucleate boiling, f_s. This was done since it was possible to approximate f_a directly from the fuel surface temperature and the assumption that the gross temperature distribution over the core was proportional to the bare core power distribution. It is expected that the final factor, f_s, will be independent of the pulse parameters for a particular system and that it will prove essentially independent of the reactor parameters in any heterogeneous water moderated system. In effect, the final parameter, f_s, was left to be calibrated by any particular pulse. The test of this model was of course a constant f_s. For the two boiling runs considered, the values of f_s calculated were 3.7×10^{-3} and 3.3×10^{-3} which differ

by less than 2%. The 2% difference is less than would be expected for the accuracy of the input data. The reactivity effect of the steam is then

$$\Delta k_s(t) = \left(\frac{-7.28 \times 10^{-6}}{1.35 \times 10^{-6}} \Delta k/\text{MW-sec}\right) (f_a f_s \; E_s(t)) \tag{41}$$

$$= -1.87 \times 10^{-2} \; f_a E_s(t)$$

f_a is fraction of core heat transfer area involved in boiling,

f_s is fraction of energy actually forming steam,

and $E_s(t)$ is total energy into core after initial boiling in MW-sec.

The fraction f_a was calculated assuming the gross core temperature distribution had reached a dynamic equilibrium with the power and the power distribution could be approximated by that of an equivalent bare core with an effective height, $2Z_e$, and radius, R_e. The gross temperature distribution in the core is then

$$\theta\,(r,Z) = \theta_o \, J_o\left(\frac{2.4048\, r}{R_e}\right) \cos \frac{\pi Z}{2Z_e} \tag{42}$$

where θ_o is the surface temperature at the time of interest on the axial center line of a central fuel element. The maximum value of Z, Z_{max}, for which boiling will occur on any plate can then be obtained knowing the boiling temperature, θ_b, and the axial centerline surface temperature,

$$\theta_o \; J_o\left(\frac{2.4048r}{R_e}\right).$$

That is

$$\cos \frac{\pi Z_{max}}{2\, Z_e} = \frac{\theta_b}{\theta_o \; J_o\left(\frac{2.4048r}{R_e}\right)} \tag{43}$$

so that

$$\frac{Z_{max}}{Z_e} = \frac{2}{\pi} \cos^{-1} \left(\frac{\theta_b}{\theta_o \; J_o(\frac{2.4048r}{R_e})} \right) \quad .$$

The maximum value of r for which boiling will occur, r_{max}, on the core axial centerline can be obtained from

$$\theta_b = \theta_o \; J_o \left(\frac{2.4048 r_{max}}{R_e} \right)$$

so that (44)

$$\frac{r_{max}}{R_e} = \frac{1}{2.4048} \; J_o \left(\frac{\theta_b}{\theta_o} \right)$$

where $y = J_o^{-1}(x)$ is the inverse of $x = J_o(y)$.

The volume fraction of the core having a fuel surface temperature above the boiling temperature is

$$f_v = \frac{1}{\pi R_e^2 \; (2Z_e)} \int_o^{r_{max}} 2 \; Z_{max} \; 2\pi r dr$$

$$= \frac{2}{R_e^2} \int_o^{r_{max}} \frac{Z_{max}}{Z_e} \; r dr$$

Making the change in variable $\eta = r/R_e$, the integral becomes

$$f_v = 2 \int_o^{\eta_{max}} \frac{Z_{max}(\eta)}{Z_e} \; \eta d\eta \quad . \qquad (46)$$

Substituting the value of Z_{max} / Z_e from equation (43) yields

$$f_v = 4/\pi \int_o^{\eta_{max}} \cos^{-1} \frac{\theta_b}{\theta_\infty J_o(2.4048 \eta)} \; \eta d\eta \quad . \qquad (47)$$

This integration was then carried out numerically using the value of $\eta_{max} = r_{max} / R_e$ determined from equation (44). Since there is a

constant heat transfer area per unit volume in the core then $\underline{f_v} = \underline{f_a}$. For the two boiling runs $\underline{\tau} = 15.8$ msec and 23 msec, $\underline{f_a}$ was equal to 0.116 and 0.084, respectively.

The total energy transferred to the moderator after the time of initial boiling was obtained by considering the moderator volume associated with unit surface area in the central fuel element. The heat content of the moderator at the time boiling temperatures were reached at the surface and at the time of peak power were calculated based on the conduction model. While this model gave a somewhat erroneous temperature distribution above boiling temperatures it accurately represented the heat flow into the moderator. The difference between the moderator heat content at the time of interest and at the time boiling temperatures were first reached was the energy available for boiling per unit fuel surface area. Plots of the central fuel surface temperature and the average moderator temperature in a central element used to calculate the moderator heat contents are shown in Figures 2 and 3. The total energy available for steam formation is obtained by multiplying by the total heat transfer surface area of the core. This somewhat overestimates the total energy but is probably the best estimate of the energy of interest since the boiling region is confined to a rather small central portion of the core. Therefore

$$E_s(t) = (\overline{\theta}_m(t) - \overline{\theta}_m(t_b))\, C_p M$$

where C_p is the heat capacity of the moderator
M is the mass of the moderator in the system
$\overline{\theta}_m(t)$ is the average moderator

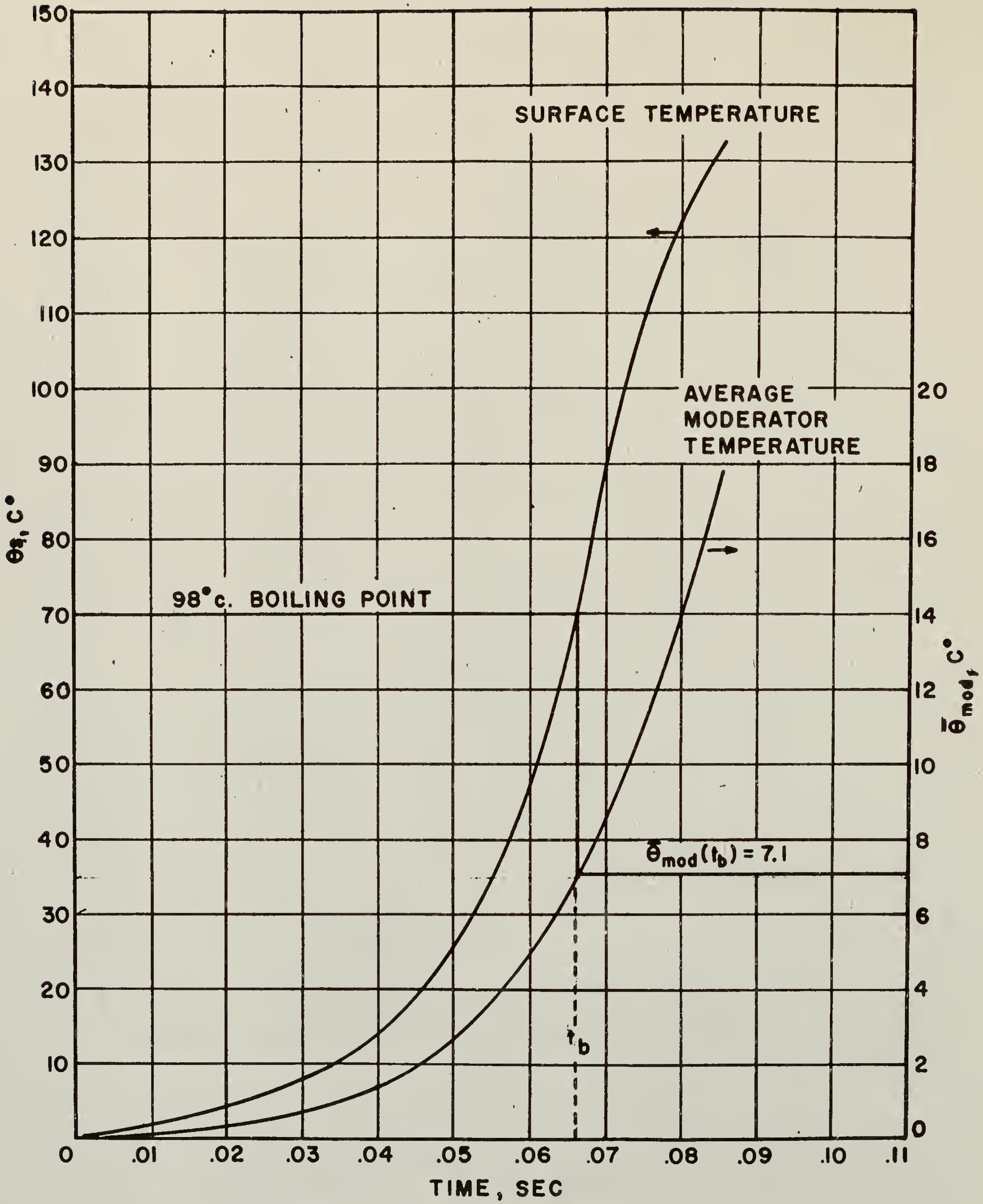

Figure 2. Graph used to determine the moderator energy for boiling calculations during a transient with an initial period of 15.8 msec.

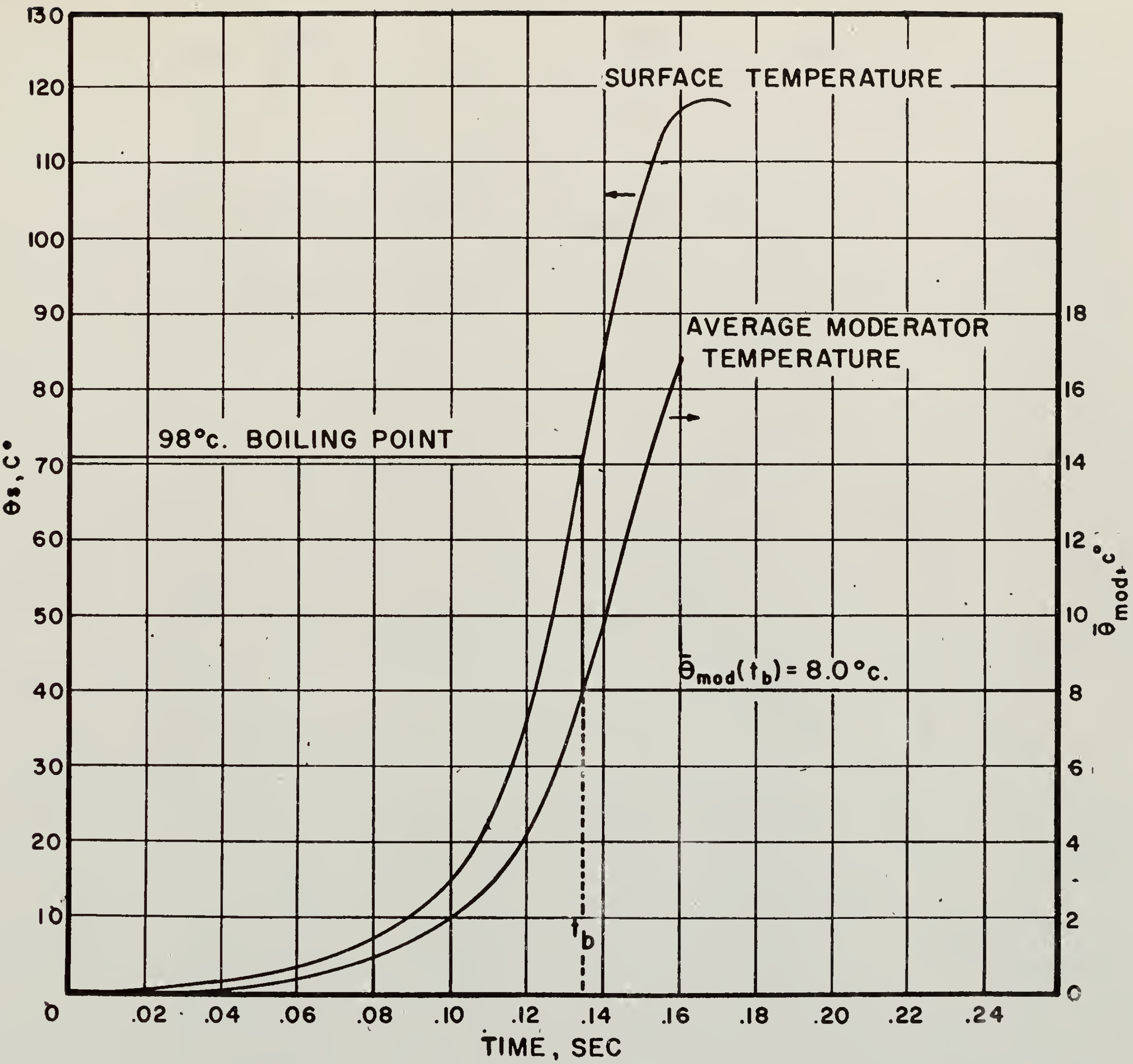

Figure 3. Graph used to determine the moderator energy content for boiling calculations during a transient with an initial period of 23 msec.

3.0 RESULTS AND DISCUSSION

3.1 Temperature Distribution and Surface Heat Flow

The temperature distributions as obtained from equations 15 and 16 are shown in Figures 4 through 7. These plots were obtained using experimental data from four transient tests on the SPERT I-A (17/28) reactor having initial periods, τ, of 15.8, 23, 120, and 150 msec, respectively. The transient burst for τ equal 15.8 and 23 msec show regions in the moderator which have temperatures above the boiling point at a pressure of one atmosphere. It is not believed that this superheating takes place. These temperature distributions are shown since such a small portion of the moderator is above the saturation point that it is not likely that it will materially affect the temperature in the remainder of the moderator or the average moderator temperature. One possibility which must be considered in calculating the reactivity effects if that pressure transients are developed which raise the boiling point above the temperatures observed in the core. This appears not to be the case for two reasons. First, experimental measurements of the pressures do not indicate sufficient rises in pressure and second, the total reactivity compensations at peak power indicate that boiling must have taken place.

The experimental surface temperature traces and the approximate analytical fits for the four transient tests are shown in Figures 8 through 11. The parameters for the analytical fits, $\theta(L,t) = \sum_{i=1}^{p} B_i \cos \beta_i t$, are shown in Table 2. The experimental power traces, actually $H_{of}(t)$, and the approximate analytical fits for the same four transient tests

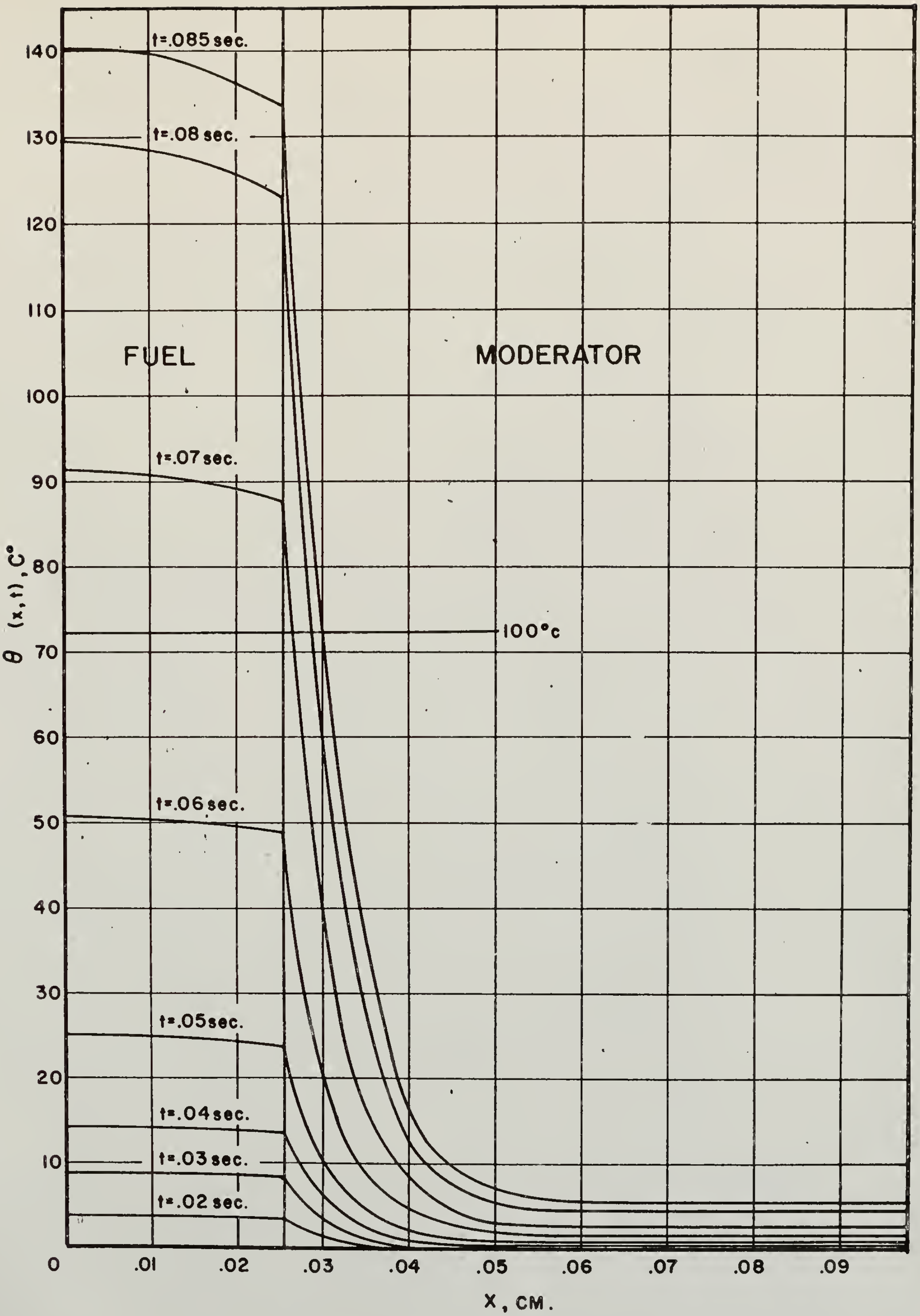

Figure 4. Temperature distributions, $\theta(x,t)$, vs position in fuel and moderator based on pure conduction during a transient with an initial period of 15.8 msec.

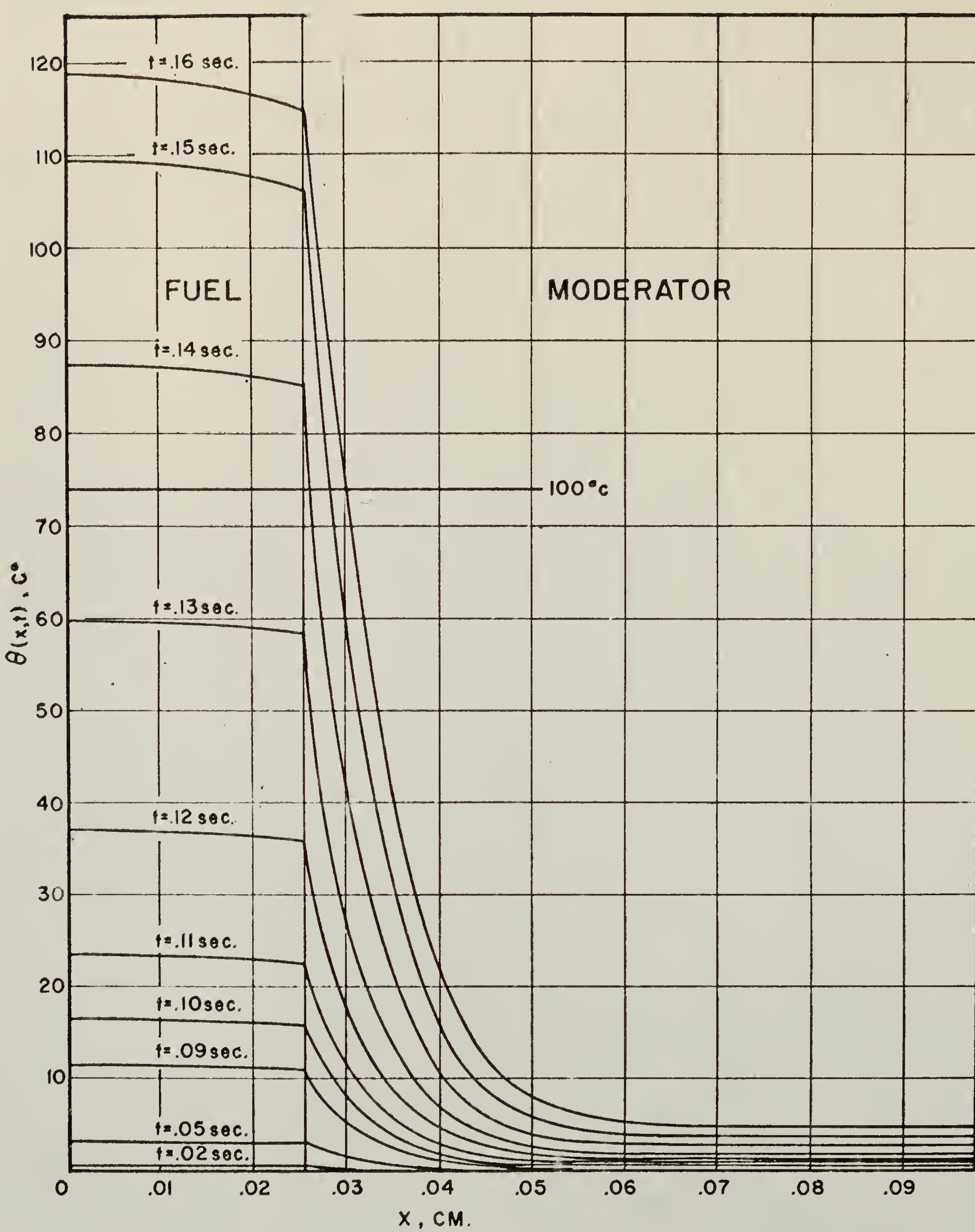

Figure 5. Temperature distributions, $\theta(x,t)$, vs position in fuel and moderator based on pure conduction during a transient with an initial period of 23 msec.

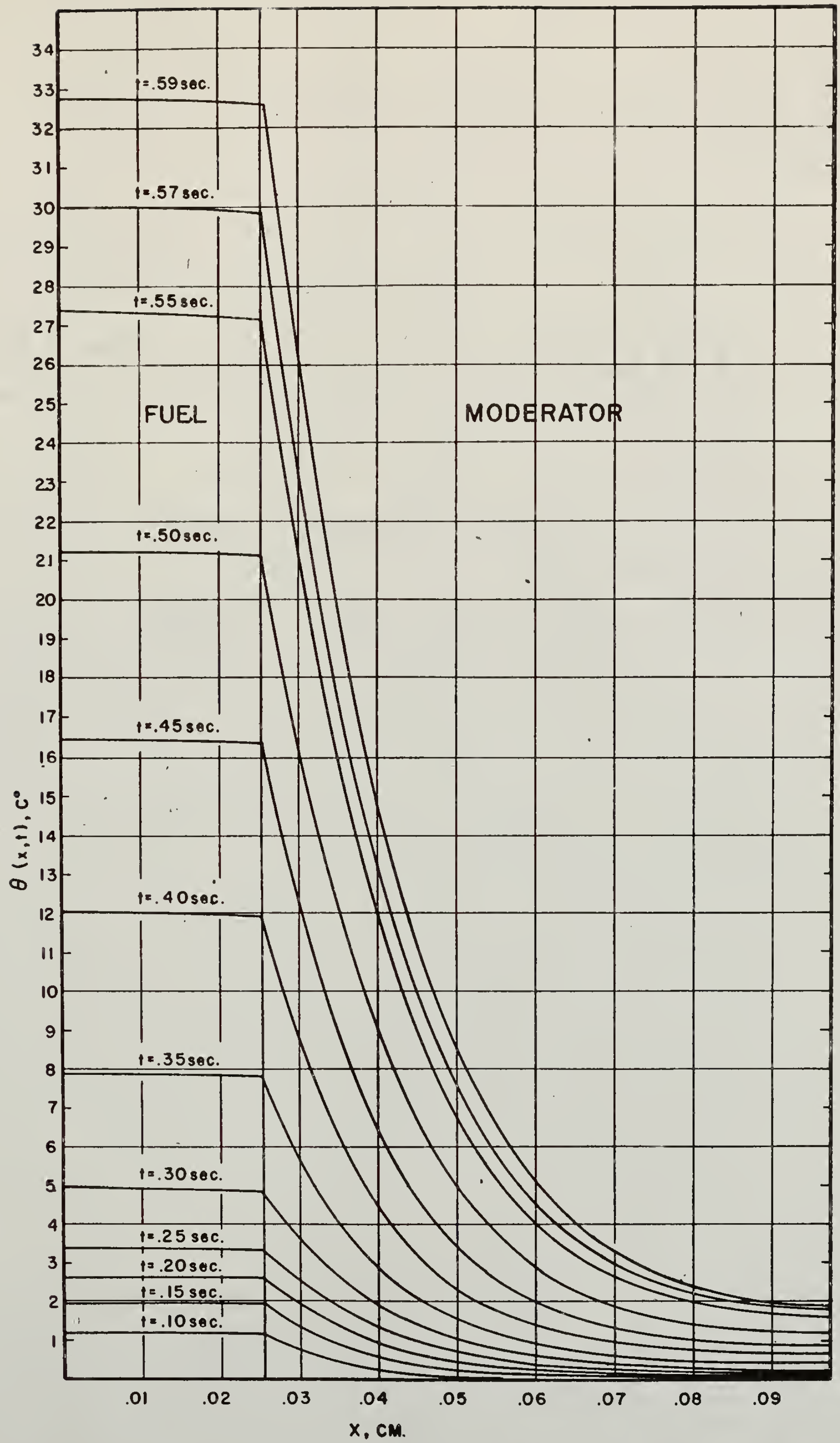

Figure 6. Temperature distributions, $\theta(x,t)$, vs position in fuel and moderator based on pure conduction during a transient with an initial period of 120 msec.

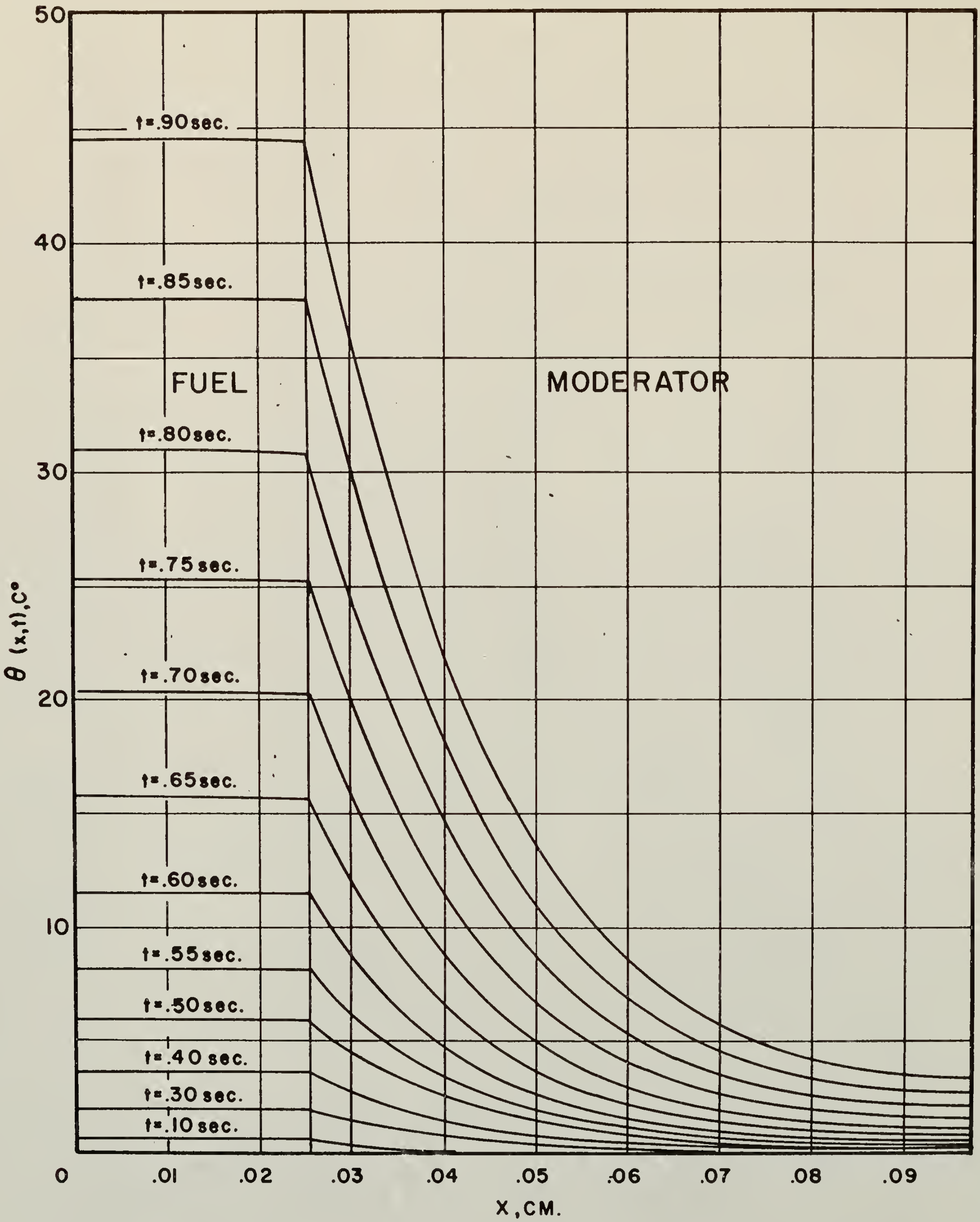

Figure 7. Temperature distributions, θ (x,t), vs position in fuel and moderator based on pure conduction during a transient with an initial period of 150 msec.

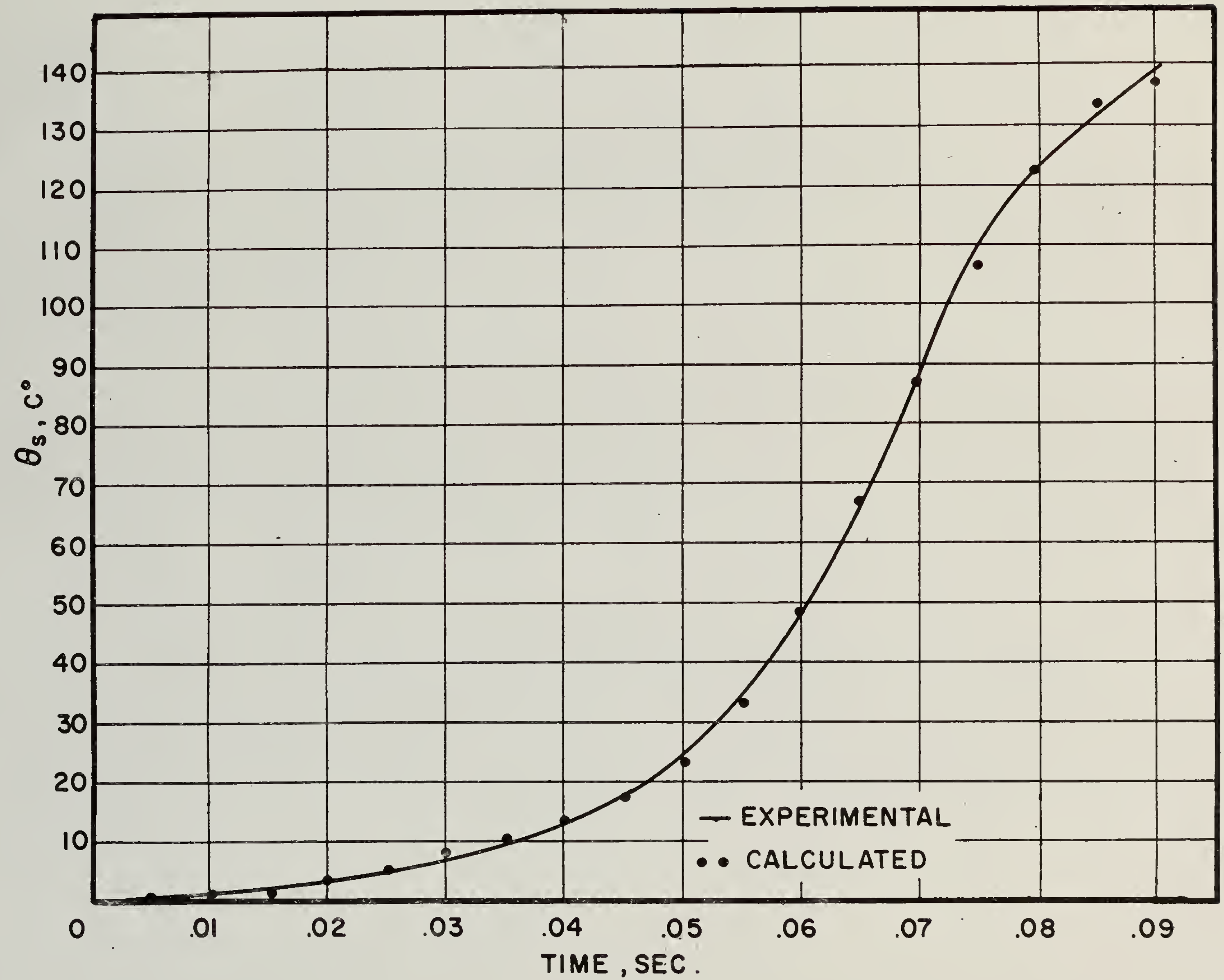

Figure 8. Interface temperatures, θ_s, vs arbitrary time during a transient with an initial period of 15.8 msec.

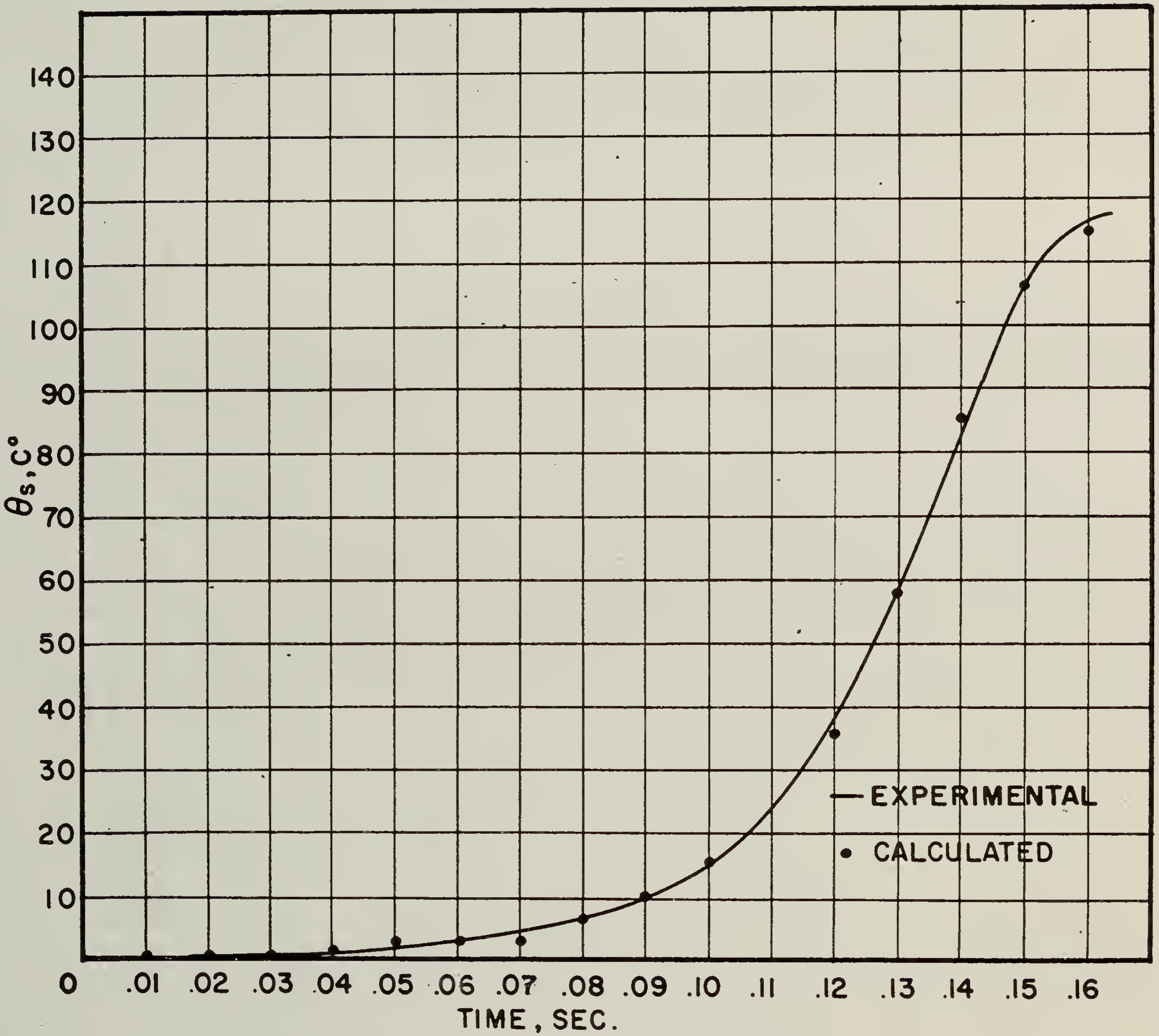

Figure 9. Interface temperatures, θ_s, vs arbitrary time during a transient with an initial period of 23 msec.

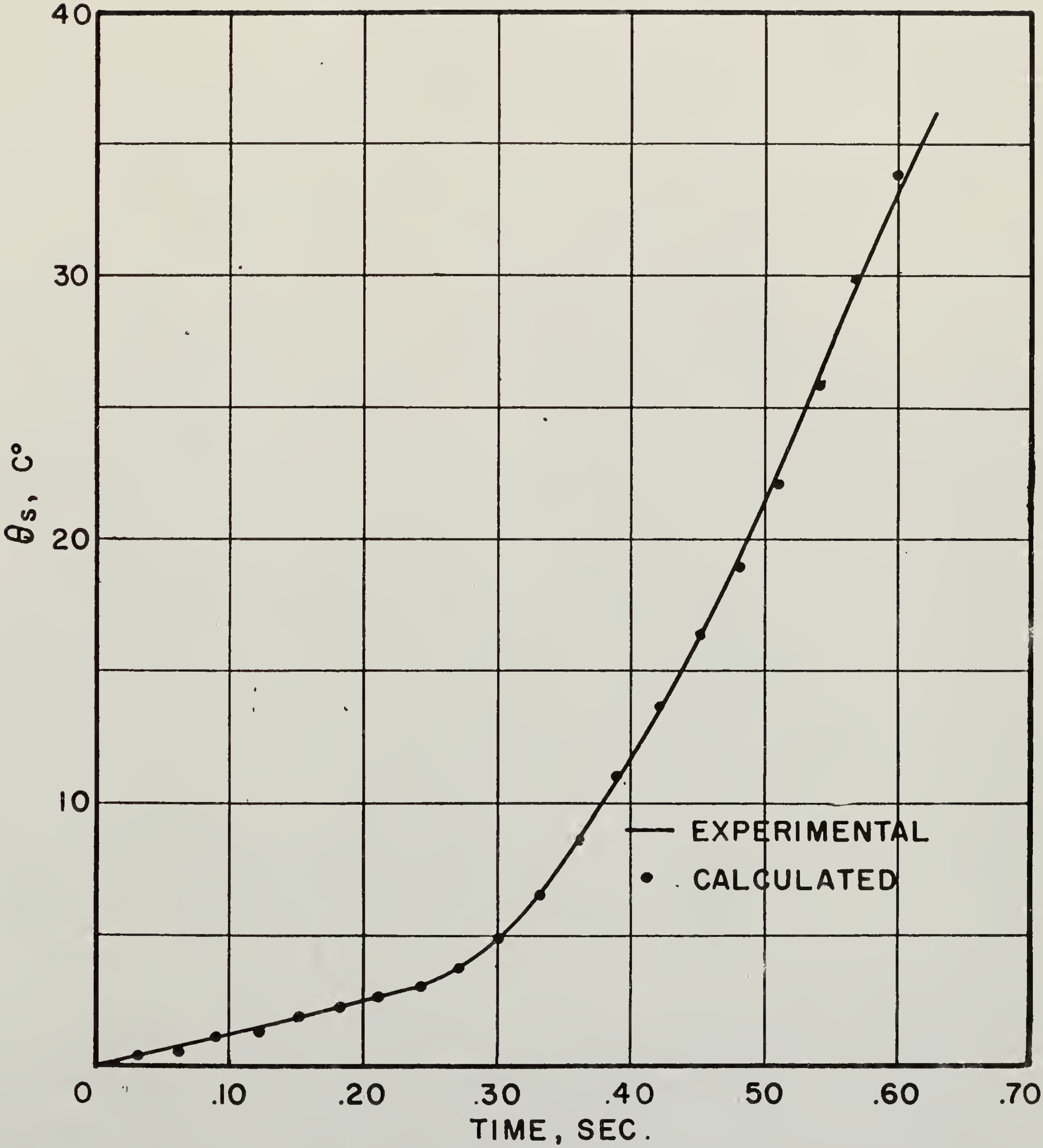

Figure 1.0. Interface temperatures, θ_s, vs arbitrary time during a transient with an initial period of 120 msec.

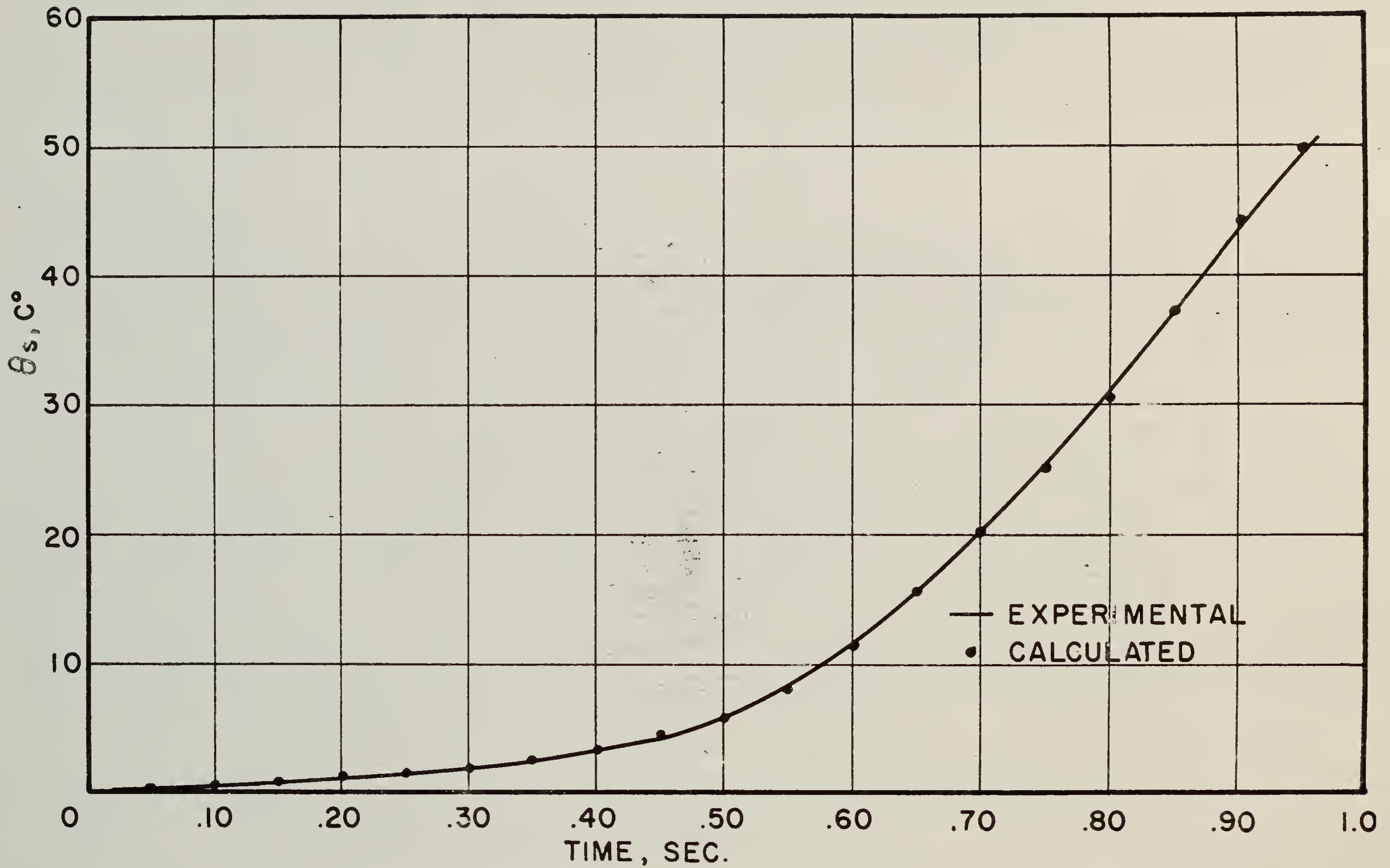

Figure II. Interface temperatures, θ_s, vs arbitrary time during a transient with an initial period of 150 msec.

Table 2. Numerical Values of Parameters for Empirical Fits of $\theta(L,t)$ Used in Equations (15) and (16)

τ, msec	B_1 / β_1	B_2 / β_2	B_3 / β_3	B_4 / β_4	B_5 / β_5	B_6 / β_6	B_7 / β_7	B_8 / β_8
150	+14.593 / 0.00	-20.958 / 3.1416	+9.474 / 6.2832	-3.763 / 9.4279	+1.313 / 12.566	-0.871 / 15.708	+0.582 / 18.850	-0.447 / 21.991
120	+12.033 / 0.00	-15.831 / 4.760	+5.875 / 9.520	-2.067 / 14.280	+6.817 / 19.040	-0.829 / 23.800	+0.399 / 28.560	-0.100 / 33.320
23	+35.752 / 0.00	-54.937 / 17.405	+26.246 / 34.810	-7.248 / 52.215	-2.118 / 69.620	+4.476 / 87.024	-3.802 / 104.429	+2.413 / 121.834
15.8	42.074 / 0.00	-59.957 / 34.906	+25.821 / 69.812	-9.279 / 104.718	+1.094 / 139.624	0.838 / 174.530	-- / --	-- / --

are shown in Figures 12 through 15. The parameters for the analytical fits, $H_{of}(t) = \sum_{j=1}^{s} A_j e^{\lambda_j t}$ are shown in Table 3.

One physical check on the solutions not required by the mathematical formulation of the problem is that the heat flow out of the fuel must equal the heat flow into the moderator. The heat flow data are shown in Figures 16 through 19. It is obvious from inspection of these data that the equivalent heat flow condition is not well satisfied. There are several possible explanations for this discrepancy. First, the heat flow data is somewhat more sensitive to the analytical fits of the surface temperature and the heat generation rates than the average temperatures. Second, the cladding between the meat and the moderator was neglected in calculating the temperature distributions. Again, the average temperatures are far less sensitive to this approximation than the heat flow calculations. Finally, it can be seen by investigating the heat flow equations that the discrepancies could be decreased by introducing a positive phase angle to the surface temperature fits. This could be attributed to a delay time in the surface temperature measurements.

The average temperatures in the fuel and moderator as a function of time were calculated by means of a numerical integration of the calculated temperature distributions. These data are given in Tables 4 through 7.

3.2 Reactivity Effects

The reactivity compensations

$$\Delta k_c(t) = \Delta k(o) - \Delta k(t)$$

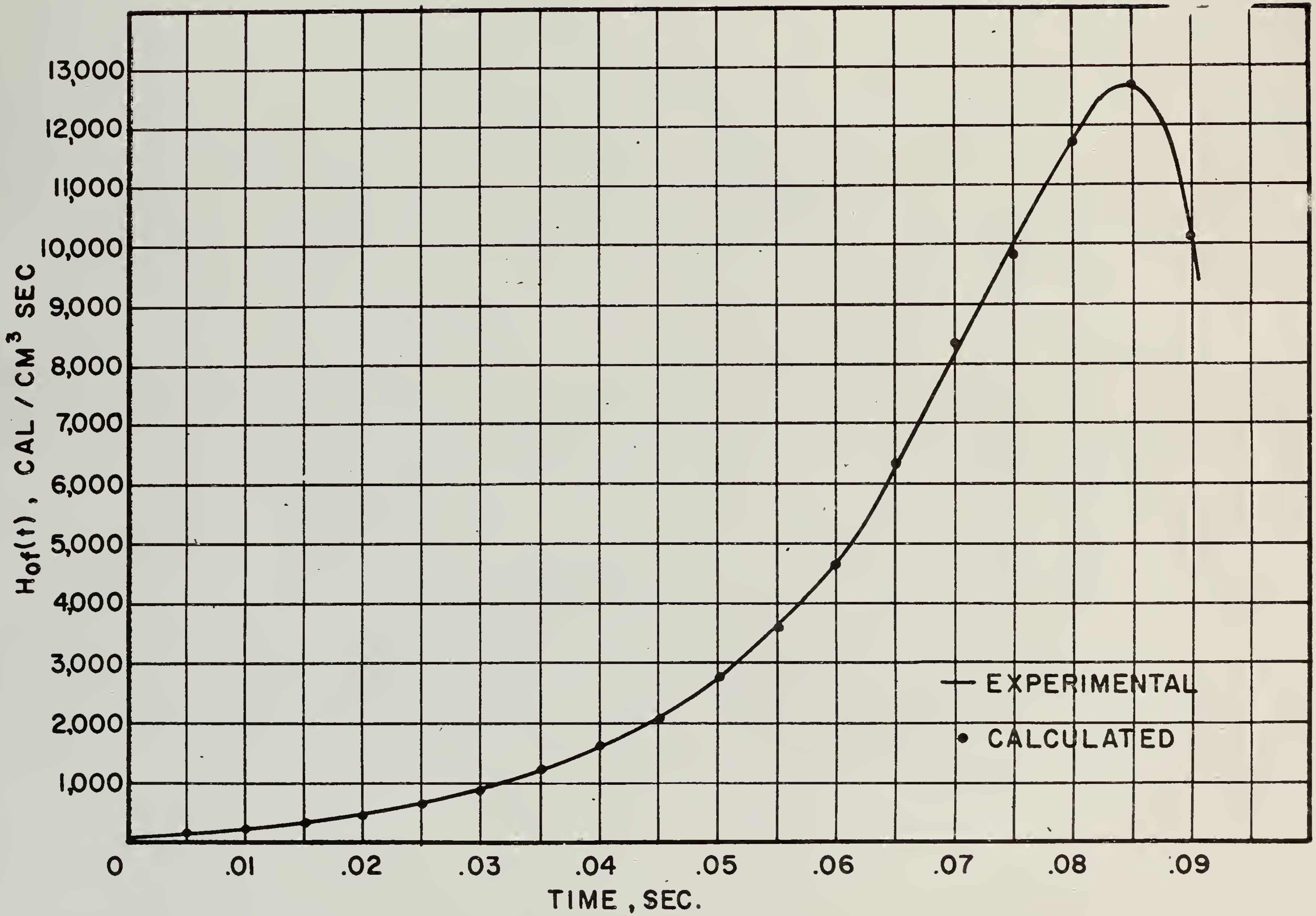

Figure 12. Internal plate heat generation rate, $H_{of}(t)$, vs arbitrary time during a transient with an initial period of 15.8 msec.

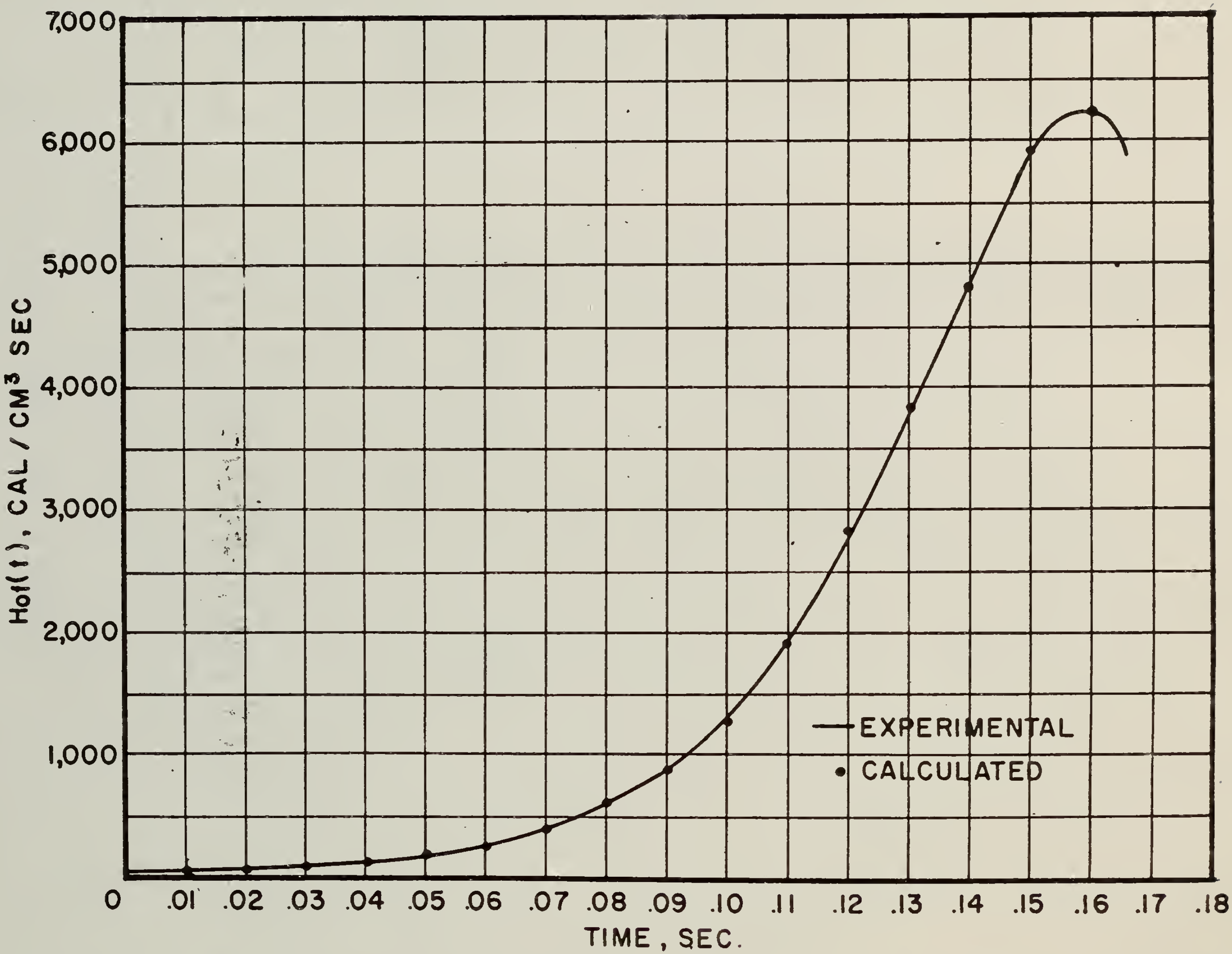

Figure 13. Internal plate heat generation rate, $H_{of}(t)$, vs arbitrary time during a transient with an initial period of 23 msec.

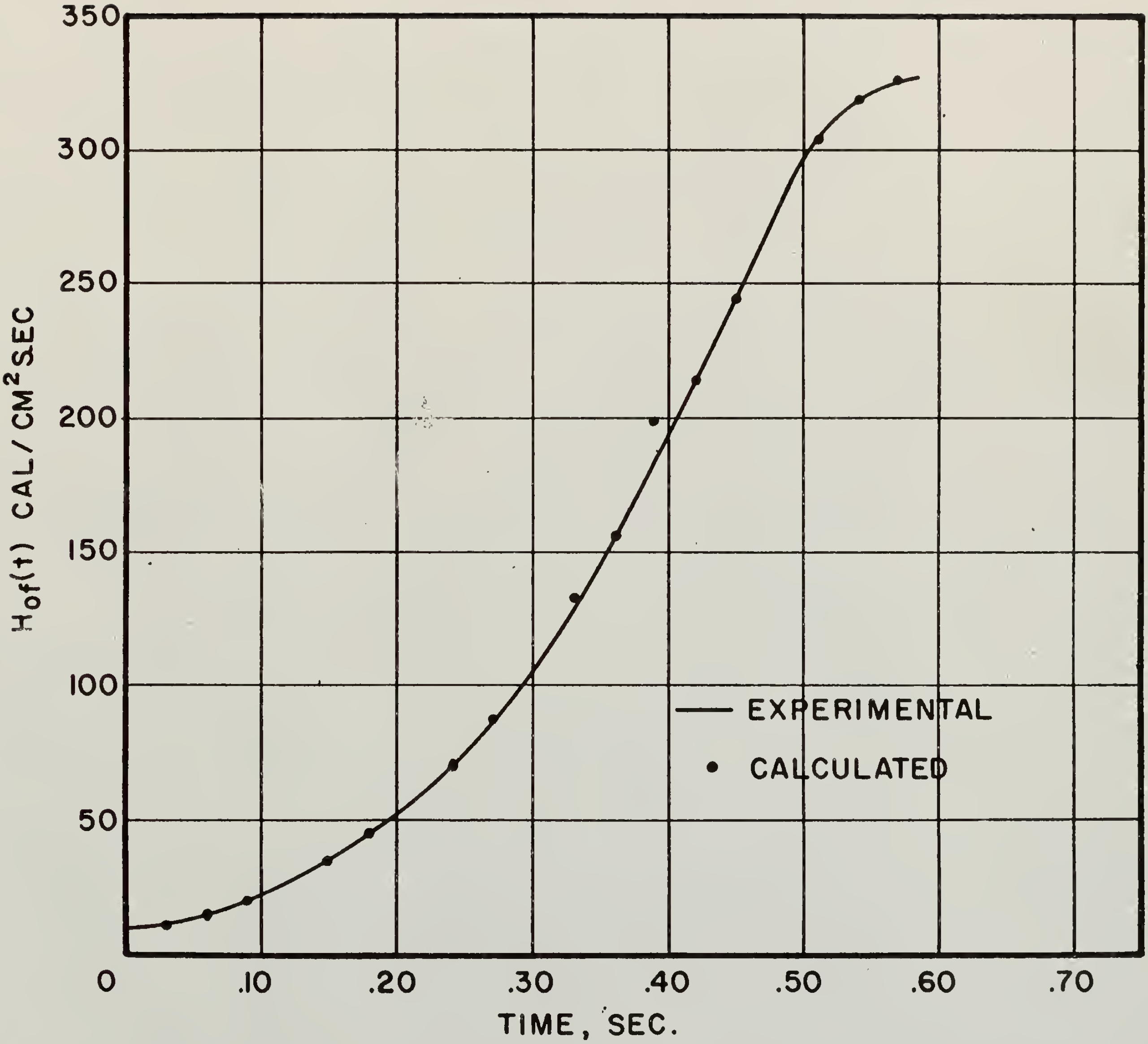

Figure 14. Internal plate heat generation rate, $H_{of}(t)$, vs arbitrary time during a transient with an initial period of 120 msec.

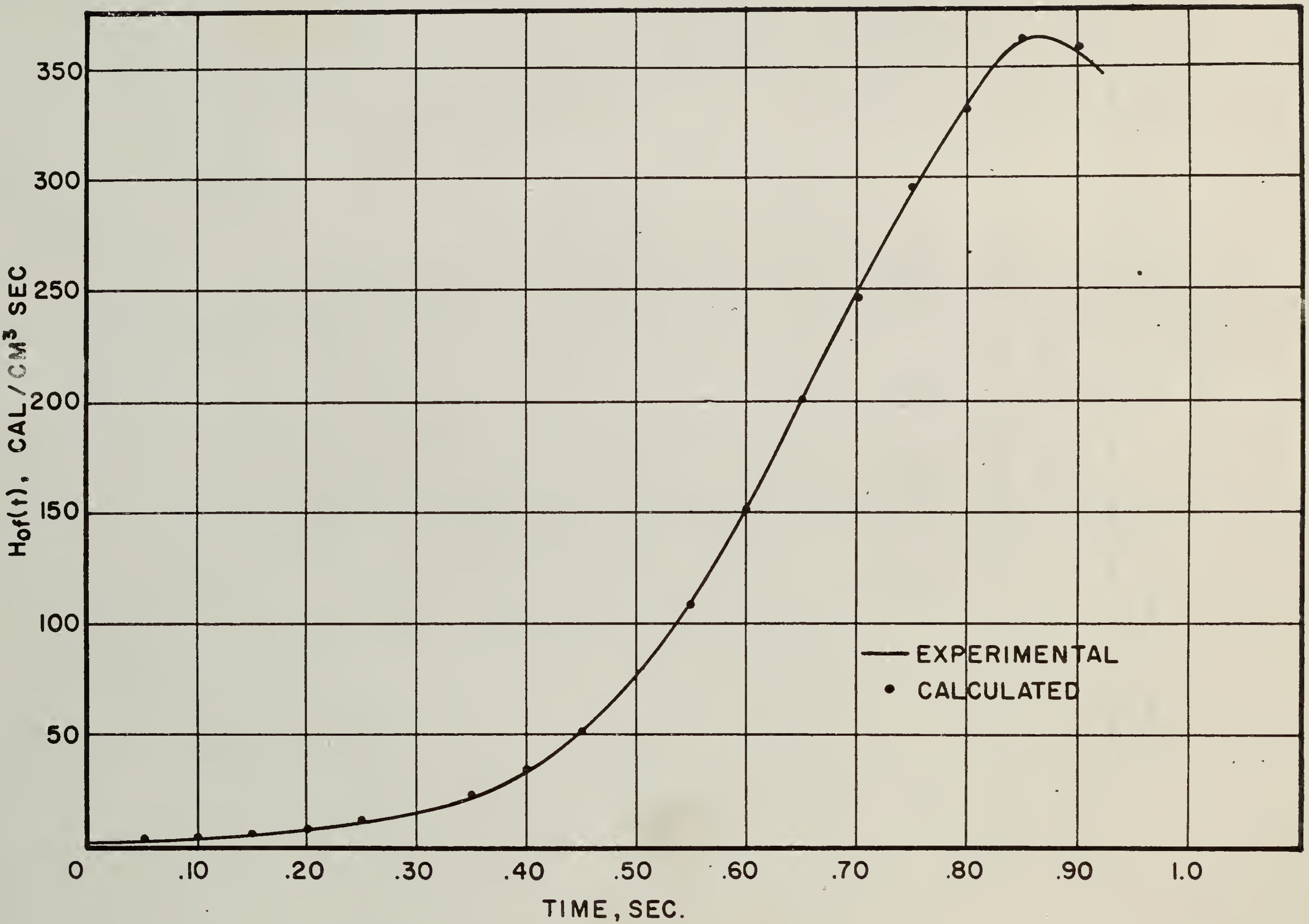

Figure 15. Internal plate heat generation rate, $H_{of}(t)$, vs arbitrary time during a transient with an initial period of 150 msec.

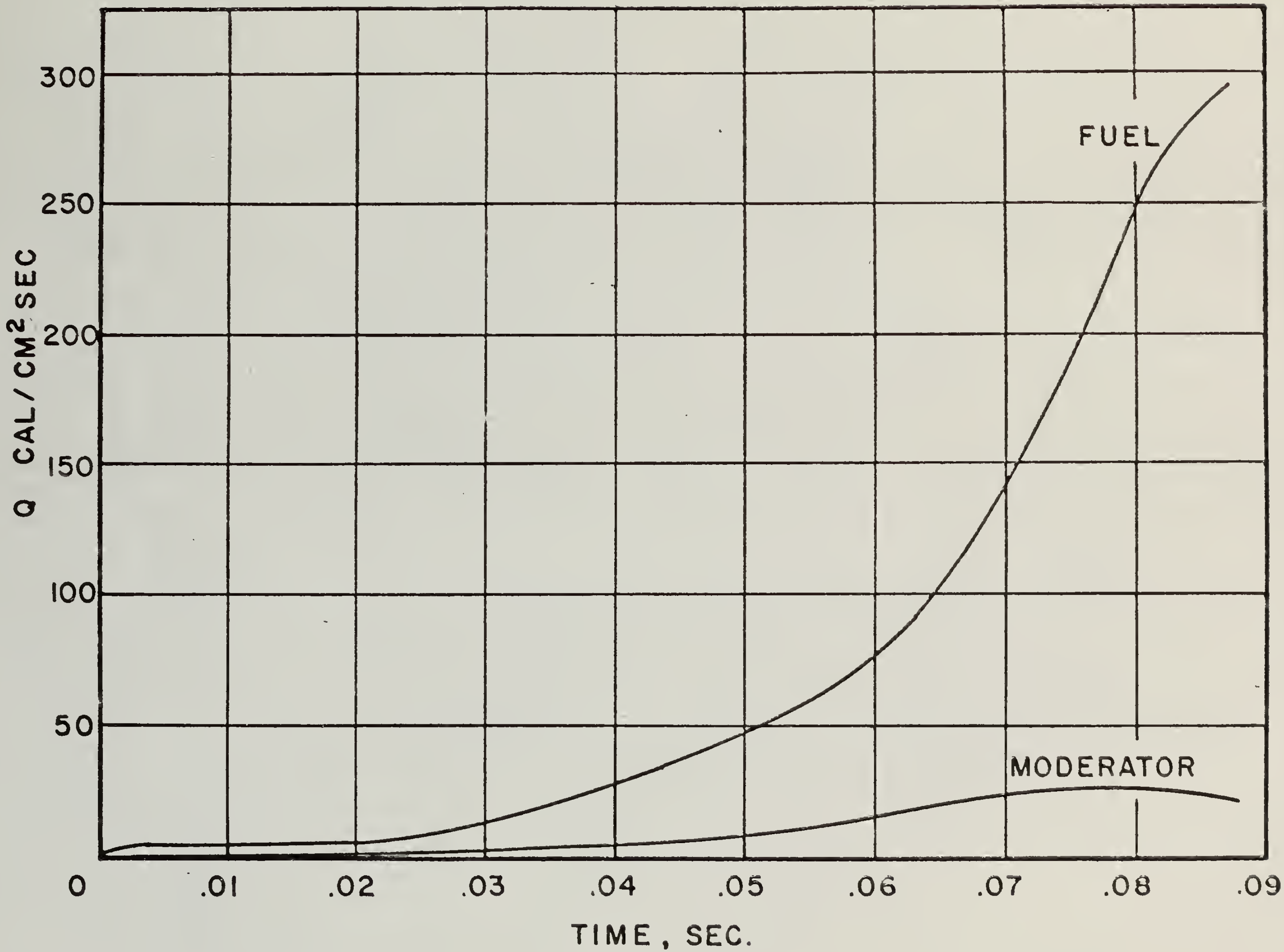

Figure 16. Comparison ot, Q, heat flow rates per unit area out of the fuel and into the moderator vs arbitrary time during a transient with an initial period of 15.8 msec.

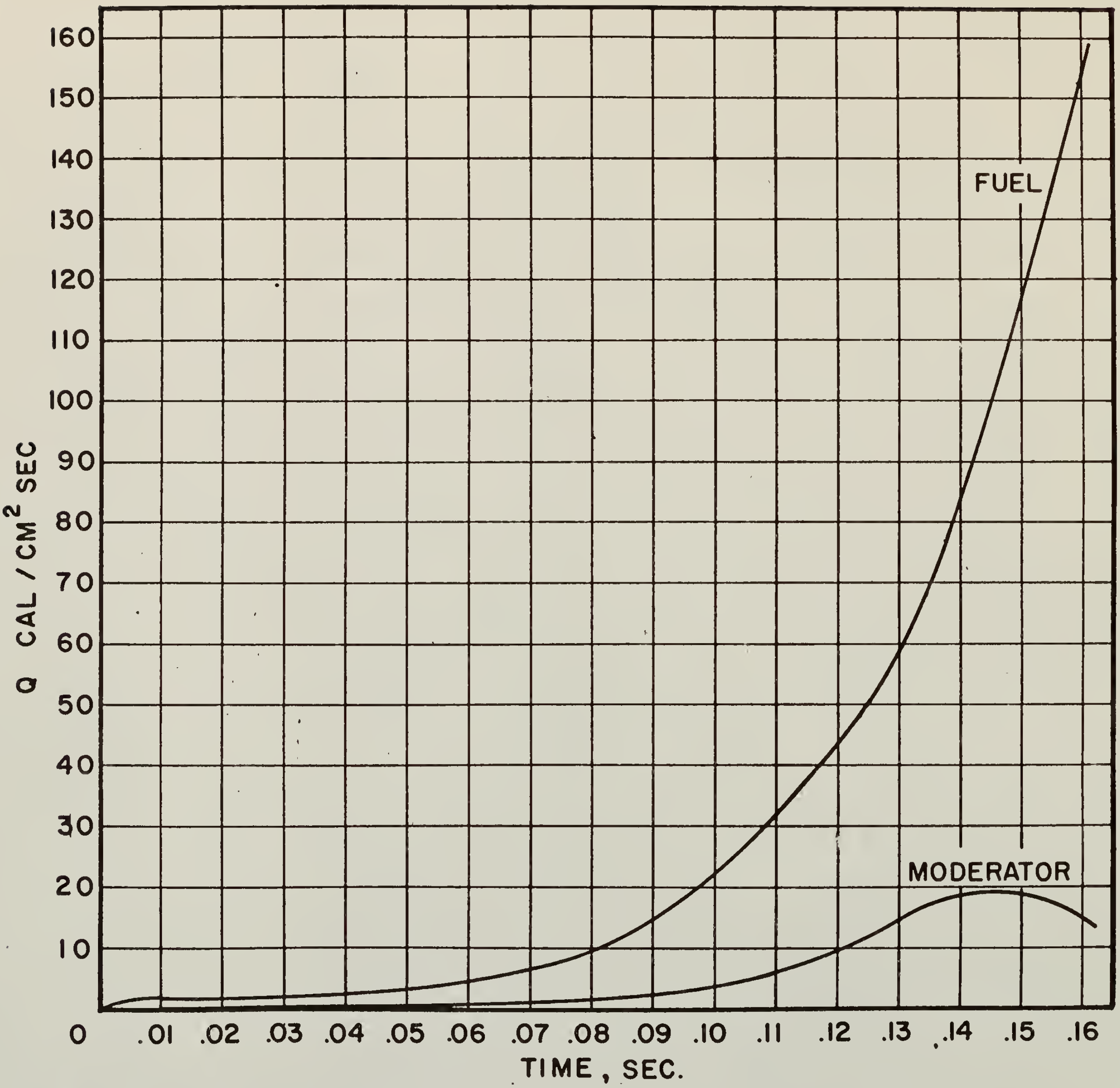

Figure 17. Comparision of, Q, heat flow rates per unit area out of the fuel and into the moderator vs arbitrary time during a transient with an initial period of 23 msec.

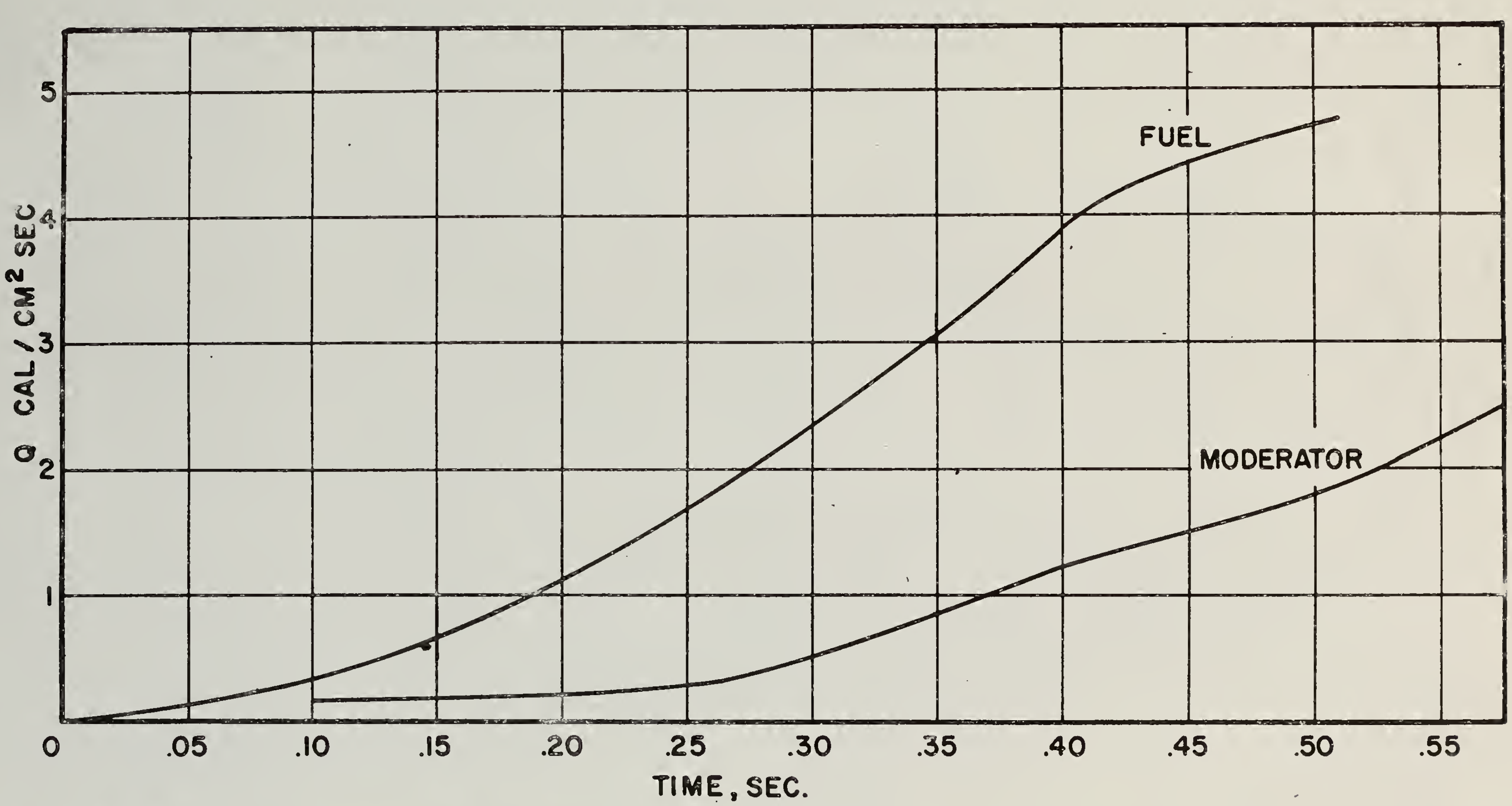

Figure 18. Comparison of, Q, heat flow rates per unit area out of the fuel and into the moderator vs arbitrary time during a transient with an initial period of 120 msec.

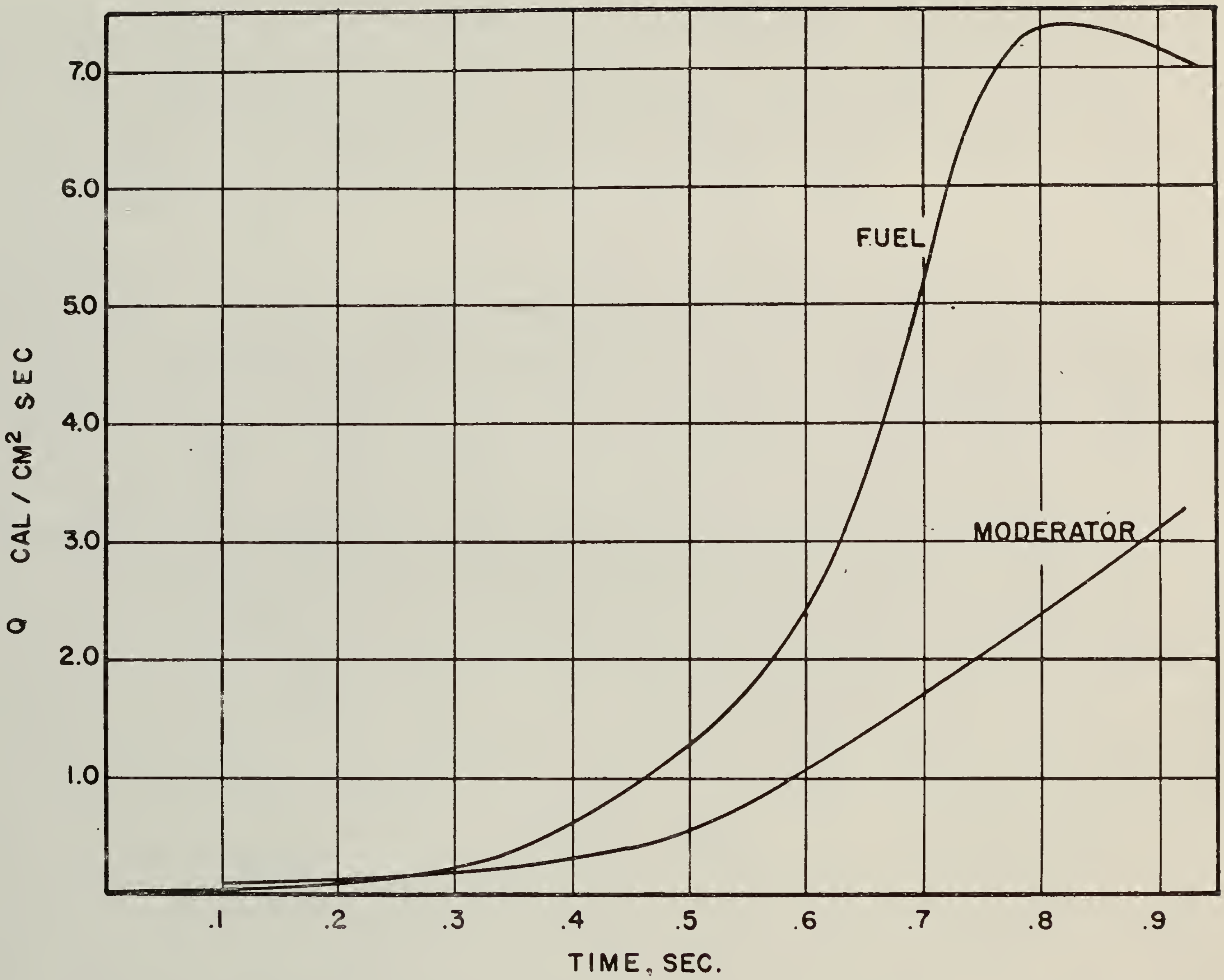

Figure 19. Comparision of, Q, heat flow rates per unit area out of the fuel and into the moderator vs arbitrary time during a transient with an initial period of 150 msec.

Table 3. Numerical Values of Parameters for Empirical Fits of $H_{of}(t)$ Used in Equations (15) and (16)

τ, msec	A_1	λ_1	A_2	λ_2	A_3	λ_3	A_4	λ_4	A_5	λ_5
150	+1.912	7.426	-2.097×10^{-5}	21.090	$+6.062\times10^{-8}$	27.260	---	---	---	---
120	9.218	9.204	-6.486×10^{-2}	20.233	$+3.564\times10^{-4}$	30.093	-1.856×10^{-7}	42.208	$+3.019\times10^{-19}$	85.147
23	2.414×10^{1}	39.761	-4.364×10^{-5}	124.03	9.563×10^{-10}	187.61	---	---	---	---
15.8	1.364×10^{2}	64.573	-2.993	104.11	---	---	---	---	---	---

Table 4. Average Temperature Rise in the Fuel and Moderator for τ = 15.8 msec.

t, sec.	$\bar{\theta}_{fuel}(t)$ °C	$\bar{\theta}_{mod}(t)$, °C	$(\bar{\theta}_{fuel}-\bar{\theta}_{mod})$, °C
.085	138.53	16.732	121.80
.080	127.34	13.99	113.35
0.075	110.34	10.81	99.53
.070	89.99	8.664	81.33
.060	50.12	4.87	45.25
.050	24.58	2.446	22.13
.040	13.91	1.333	12.58
.030	8.51	0.692	7.82
.020	3.52	.271	3.25
.010	1.039	.0767	.962

Table 5. Average Temperature Rise in the Fuel and Moderator for τ = 23 msec.

t, sec.	$\bar{\theta}_{fuel}(t)$, °C	$\bar{\theta}_{mod}(t)$, °C	$(\bar{\theta}_{fuel}-\bar{\theta}_{mod})$, °C
.16	117.76	16.832	100.93
.15	108.71	13.371	95.34
.14	86.92	9.764	77.16
.13	59.48	6.459	53.02
.12	36.58	4.218	32.35
.11	23.17	2.759	20.41
.10	16.28	1.869	15.21
.09	11.25	1.231	10.02
.05	3.13	.260	2.87

Table 6. Average Temperature Rise in the Fuel and Moderator for τ = 120 msec.

t,sec.	$\bar{\theta}_{fuel}(t)$,°C	$\bar{\theta}_{mod}(t)$,°C	$(\bar{\theta}_{fuel}-\bar{\theta}_{mod})$,°C
.59	32.66	8.572	24.09
.57	29.95	7.738	22.21
.55	27.33	6.935	20.39
.50	21.15	5.212	15.94
.45	16.40	3.834	12.57
.40	11.98	2.660	9.32
.35	7.84	1.744	6.10
.30	4.89	1.143	3.75
.20	2.59	.534	2.06
.10	1.17	.164	1.01

Table 7. Average Temperature Rise in the Fuel and Moderator for τ = 150 msec.

t,sec.	$\bar{\theta}_{fuel}(t)$,°C	$\bar{\theta}_{mod}(t)$,°C	$(\bar{\theta}_{fuel}-\bar{\theta}_{mod})$,°C
.90	44.46	12.994	31.47
.85	37.49	10.630	26.86
.80	30.87	8.510	22.36
.75	25.23	6.678	18.55
.70	20.31	5.111	15.20
.65	15.70	4.644	11.06
.60	11.48	2.750	8.73
.55	8.11	1.989	6.12
.50	4.86	1.472	4.39
.40	3.58	.834	2.75
.30	1.91	.431	1.48
.10	.602	.0652	.537

where $\underline{\Delta k(o)}$ is the initial reactivity insertion to start the transient and $\underline{\Delta k(t)}$ is the excess reactivity of the system at any time, $\underline{t}$, are shown as a function of time in Tables 8 through 11. Tables 8 through 11 also show the components of the compensated reactivity due to the temperature coefficient $\underline{\Delta k_T(t)}$, fuel plate expansion $\underline{\Delta k_E(t)}$ and steam formation, $\underline{\Delta k_s(t)}$. The excess reactivity $\underline{\Delta k(t)}$ for two of the transients, $\underline{\tau}$ equals 120 and 150 msec respectively, is compared with equivalent data obtained from a kinetic analysis of the power burst shapes by Miller (34) in Figures 20 and 21. The reactivity compensation at peak power, is shown along with comparable data from the kinetic analysis in Figure 22. Figure 22 also includes the components of the reactivity compensations for a model suggested by S. G. Forbes (15) as well as for the model suggested in this report.

3.3 Conclusions

The forms of all of the solutions shown in equations (9) through (16) are such that three terms are developed. The first term represents the steady state solution resulting from the surface temperature boundary condition. The second term includes the transient portion of both the surface temperature boundary condition and the forcing function, the heat generation rate. The final term represents the steady state or equilibrium solution resulting from the forcing function. It has been common in several previous works (27,36) to assume that the temperature is separable in space and time. It can be seen from the derived solutions that this will be a good approximation of the temperature distribution only when the second term, the transient solution, has died out. The

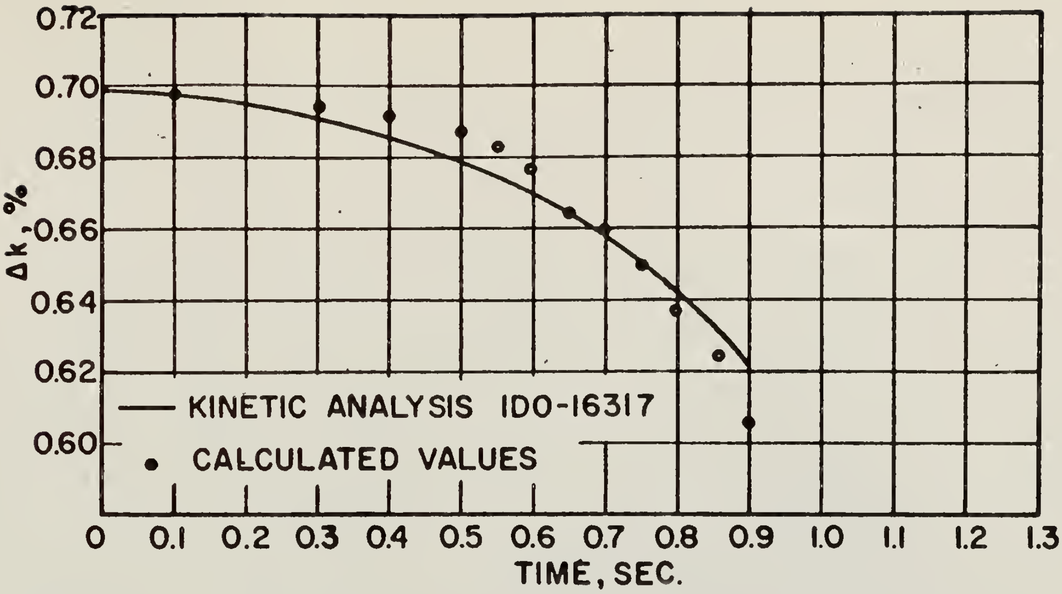

Figure 20. Comparison of calculated percent excess reactivities, Δk, and those obtained by kinetic analysis vs arbitrary time during a transient with an initial period of 120 msec.

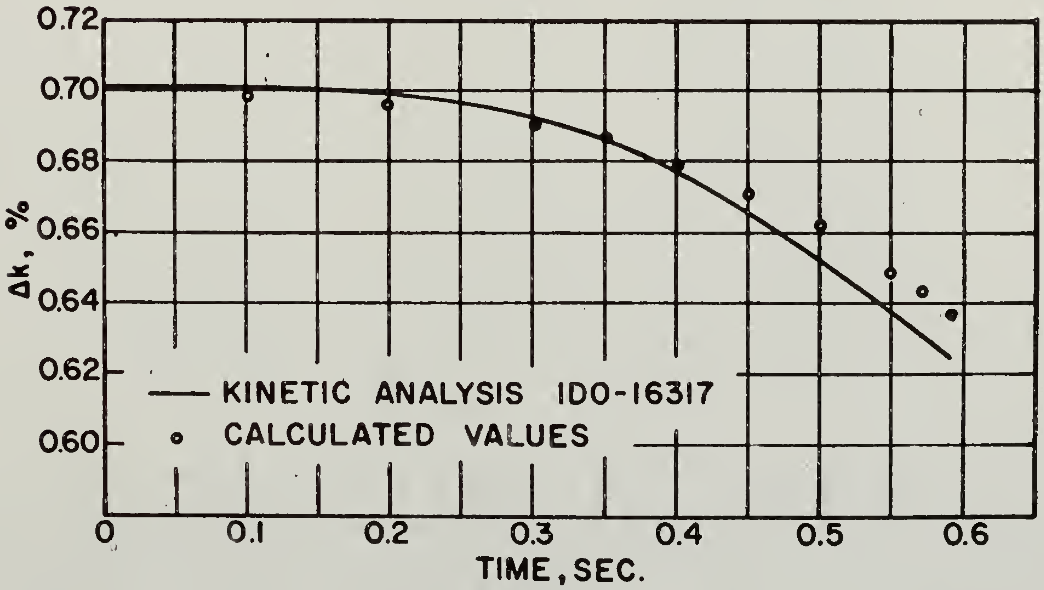

Figure 21. Comparison of calculated percent excess reactivities, Δk, and those obtained by kinetic analysis vs arbitrary time during a transient with an initial period of 150 msec.

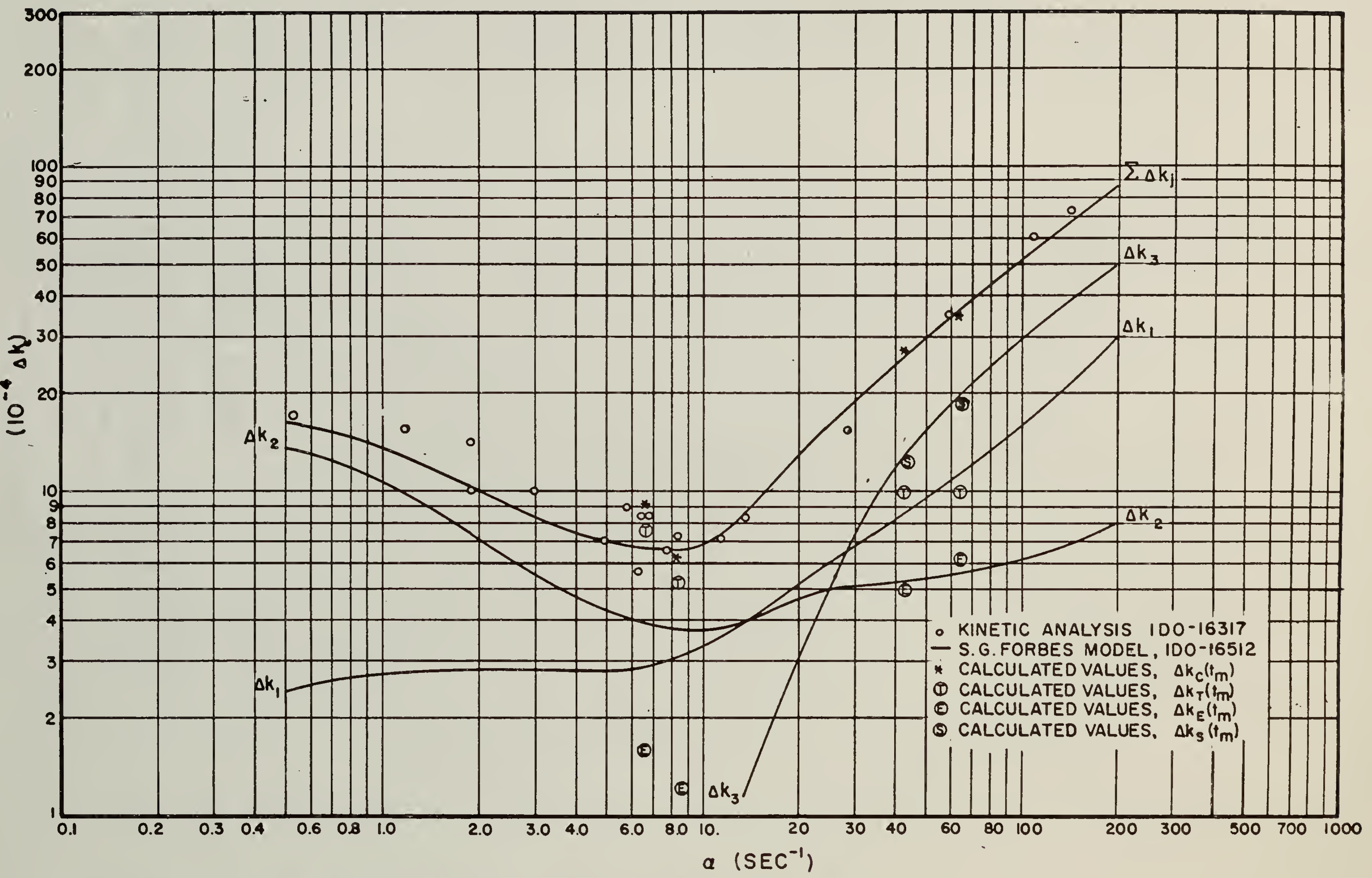

Figure 22. Peak power compensated reactivities, Δk_C, vs reciprocal period, α.

Table 8. Compensated Reactivities for τ = 15.8 msec Run

t, sec	$\Delta k_T(t)$	$\Delta k_E(t)$	$\Delta k_S(t)$	$\Delta k_C(t)$
0.084	0.979×10^{-3}	0.609×10^{-3}	1.86×10^{-3}	3.45×10^{-3}
0.080	0.818×10^{-3}	0.546×10^{-3}	1.21×10^{-3}	2.60×10^{-3}
0.075	0.632×10^{-3}	0.498×10^{-3}	0.46×10^{-3}	1.59×10^{-3}
0.070	0.684×10^{-3}	0.407×10^{-3}	0.13×10^{-3}	1.22×10^{-3}
0.060	0.285×10^{-3}	0.226×10^{-3}	0.0	0.51×10^{-3}
0.050	0.143×10^{-3}	0.111×10^{-3}	0.0	0.25×10^{-3}
0.040	0.078×10^{-3}	0.063×10^{-3}	0.0	0.14×10^{-3}
0.030	0.050×10^{-3}	0.039×10^{-3}	0.0	0.08×10^{-3}
0.020	0.016×10^{-3}	0.016×10^{-3}	0.0	0.03×10^{-3}
0.010	0.004×10^{-3}	0.005×10^{-3}	0.0	0.01×10^{-3}

Table 9. Compensated Reactivites for τ = 23 msec Run

t, sec	$\Delta k_T(t)$	$\Delta k_E(t)$	$\Delta k_S(t)$	$\Delta k_C(t)$
0.16	0.985×10^{-3}	0.505×10^{-3}	1.21×10^{-3}	2.70×10^{-3}
0.15	0.782×10^{-3}	0.468×10^{-3}	0.54×10^{-3}	1.79×10^{-3}
0.14	0.571×10^{-3}	0.386×10^{-3}	0.003×10^{-3}	0.96×10^{-3}
0.13	0.378×10^{-3}	0.265×10^{-3}	0.0	0.64×10^{-3}
0.12	0.247×10^{-3}	0.162×10^{-3}	0.0	0.41×10^{-3}
0.11	0.161×10^{-3}	0.102×10^{-3}	0.0	0.26×10^{-3}
0.10	0.109×10^{-3}	0.076×10^{-3}	0.0	0.18×10^{-3}
0.09	0.072×10^{-3}	0.050×10^{-3}	0.0	0.12×10^{-3}
0.05	0.015×10^{-3}	0.014×10^{-3}	0.0	0.03×10^{-3}

Table 10. Compensated Reactivities for τ = 120 msec Run

t,sec	$\Delta k_T(t)$	$\Delta k_E(t)$	$\Delta k_S(t)$	$\Delta k_C(t)$
0.59	0.501×10^{-3}	0.120×10^{-3}	0.0	0.621×10^{-3}
0.57	0.453×10^{-3}	0.111×10^{-3}	0.0	0.564×10^{-3}
0.55	0.406×10^{-3}	0.100×10^{-3}	0.0	0.506×10^{-3}
0.50	0.305×10^{-3}	0.080×10^{-3}	0.0	0.385×10^{-3}
0.45	0.224×10^{-3}	0.063×10^{-3}	0.0	0.287×10^{-3}
0.40	0.156×10^{-3}	0.047×10^{-3}	0.0	0.203×10^{-3}
0.35	0.102×10^{-3}	0.030×10^{-3}	0.0	0.132×10^{-3}
0.30	0.067×10^{-3}	0.019×10^{-3}	0.0	0.086×10^{-3}
0.20	0.031×10^{-3}	0.010×10^{-3}	0.0	0.041×10^{-3}
0.10	0.010×10^{-3}	0.005×10^{-3}	0.0	0.015×10^{-3}

Table 11. Compensated Reactivities for τ = 150 msec Run

t,sec	$\Delta k_T(t)$	$\Delta k_E(t)$	$\Delta k_S(t)$	$\Delta k_C(t)$
0.90	0.760×10^{-3}	0.157×10^{-3}	0.0	0.915×10^{-3}
0.85	0.622×10^{-3}	0.134×10^{-3}	0.0	0.756×10^{-3}
0.80	0.498×10^{-3}	0.112×10^{-3}	0.0	0.610×10^{-3}
0.75	0.391×10^{-3}	0.093×10^{-3}	0.0	0.484×10^{-3}
0.70	0.299×10^{-3}	0.076×10^{-3}	0.0	0.375×10^{-3}
0.65	0.272×10^{-3}	0.055×10^{-3}	0.0	0.327×10^{-3}
0.60	0.161×10^{-3}	0.044×10^{-3}	0.0	0.205×10^{-3}
0.55	0.116×10^{-3}	0.031×10^{-3}	0.0	0.147×10^{-3}
0.50	0.086×10^{-3}	0.022×10^{-3}	0.0	0.108×10^{-3}
0.40	0.049×10^{-3}	0.014×10^{-3}	0.0	0.063×10^{-3}
0.30	0.025×10^{-3}	0.007×10^{-3}	0.0	0.032×10^{-3}
0.10	0.004×10^{-3}	0.003×10^{-3}	0.0	0.007×10^{-3}

numerical results obtained from evaluating the set of equations (9) and (10) and the set of equations (15) and (16) show that the transient term is negligible for all times of interest in the fuel region but it makes a significant contribution for all times of interest in the moderator.

In one sense it would be more informative to investigate the reactor burst behavior using the minimum of input data (i.e. the physical dimensions and the initial reactivity insertion) and test for corroboration of all of the experimentally measured variables. However, it seemed better in analyzing for the shutdown mechanisms to use as much of the data as possible leaving only the compensated reactivities as a check on the validity of the model. The compensated reactivity was established as a criterion because of its extreme sensitivity to the state of the system and because of its direct influence on the safety of nuclear reactors.

The results of this work are two-fold. First, a more accurate view of distribution of the energy during a transient burst is presented and second, a model based on the energy distribution was shown to predict the reactivity effects as well as any of the existing models. The advantage of this model is that assuming the fractional energy associated with void formation can be determined as the mechanism of transient boiling becomes better understood, the final empiricism can be removed from the model.

3.4 Further Investigation

There are several avenues of attack for further work in determining the inherent shutdown mechanisms. First, additional data on Spert I-A should be tested on the model proposed in this report to make certain that it is as capable of determining reactivity effects as these preliminary runs indicate. Second, application of this model to any new system

will mean that the surface temperatures and power traces would not be available. This problem can be circumvented by studying the two region conduction problem subject only to the heat generation rate forcing function. The heat generation rate can be calculated from the initial reactivity insertion, allowing a feedback from the induced negative reactivity to the heat generation rate through the reactor kinetics equations. This suggests an analog solution or possibly a digital analog combination. Third, application of the heat transfer equations developed in this report should be used to determine the mode of heat transfer during transient operation by investigating a single plate in as much detail as possible. The application of this study can probably be done more simply using electrical heating. Fourth, a detailed study of nucleate boiling at low heat fluxes is necessary before complete understanding of the mechanisms of shutdown can be obtained. This study should provide a direct measurement of the fraction of the energy used to produce steam, f_s. Fifth, work on this model should be extended to investigate further available evidence on other Spert reactors to see if it will account for changes in other parameters such as neutron life-time, pressure and coolant flow. Finally, experimental and analytical work should be done on the zirconium hydride moderated Triga systems since qualitatively they show the greatest inherent safety that has been demonstrated to date.

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APPENDICES

APPENDIX A

Derivations of Solutions for the Temperature Distribution in the Fuel and Moderator of a Unit Cell

The differential equations governing the time dependent temperature distribution in the fuel and moderator are

$$\nabla^2\theta_f(x,t) + \sum_{j=1}^{s} \frac{q_{oo}\cosh(\kappa x)a_j e^{\lambda_j t}}{k} = \frac{1}{\alpha} \frac{\partial\theta(x,t)}{\partial t} \qquad \text{(A-1)}$$

and

$$\nabla^2\theta_m(x,t) + \sum_{j=1}^{s} \frac{F A_j e^{\lambda_j t}}{k} = \frac{1}{\alpha} \frac{\partial\theta_m(x,t)}{\partial t} \qquad \text{(A-2)}$$

The differential equations, (A-1) and (A-2), are most easily solved by use of Laplace transforms. Considering the fuel region first and transforming the time variable in equation (1) yields

$$\nabla^2\Phi_f(x,S) + \sum_{j=1}^{s} \frac{q_{oo}\cosh(\kappa x)A_j}{k(S-\lambda_j)} = \frac{1}{\alpha}[S\,\Phi_f(x,S) - \theta(x,o)]. \qquad \text{(A-3)}$$

$\Phi(x,S)$ is the transform of $\theta(x,t)$, and $\theta(x,o)$ is the initial temperature distribution. Assuming the initial temperature distribution to be flat, the equation can be normalized by letting $\theta(x,o)$ equal zero. Thus $\theta(x,t)$ is the temperature excess in the fuel over the initial temperature. The initially flat temperature distribution in the fuel is unreal but it is a good approximation if the transients are started from low power levels.

Letting $\theta(x,o) = 0$ and rearranging the terms in equation (A-3) yields

$$\nabla^2\Phi_f(x,S) - \frac{S}{\alpha}\Phi_f(x,S) = -\sum_{j=1}^{s} \frac{q_{oo}\cosh(\kappa x)A_j}{k(S-\lambda_j)} \qquad \text{(A-4)}$$

The usual one-dimensional slab assumption is made and ∇^2 becomes $\frac{\partial^2}{\partial x^2}$. This assumption neglects the axial flow of heat in the fuel compared to the radial. While this might be the limiting assumption in the analysis, it is probably not seriously in error.

The homogeneous solution to the one dimensional form of equation (A-4) is well known and is derived in many standard texts. This solution is

$$\Phi_{fh}(x,S) = A \cosh \sqrt{\frac{S}{\alpha}}\, x + B \sinh \sqrt{\frac{S}{\alpha}}\, x \tag{A-5}$$

The particular solution is found by the method of undetermined coefficients and is

$$\Phi_{fp}(x,S) = \sum_{j=1}^{s} \frac{q_{oo}\, \alpha\, A_j \cosh \kappa x}{k(S-\lambda_j)(S-\alpha\kappa^2)} \tag{A-6}$$

Therefore,

$$\Phi_{fp}(x,S) = A \cosh \sqrt{\frac{S}{\alpha}}\, x + B \sinh \sqrt{\frac{S}{\alpha}}\, x + \sum_{j=1}^{s} \frac{q_{oo}\, \alpha\, A_j \cosh(\kappa x)}{k(S-\lambda_j)\,(S-\alpha\kappa_2)} . \tag{A-7}$$

The boundary conditions used to determine the constants A and B are as follows First, the temperature gradient in the center of the region (x = 0) is zero for all time. Second, the surface temperature is matched with the experimental data. The surface temperature is expressed as a sum of exponentials, $\sum_{i=1}^{p} B_i e^{\beta_i t}$ or a Fourier series, $\sum_{i=1}^{p} B_i \cos \beta_i t$. First consider the solution in which the exponential boundary condition is used.

The transformed boundary conditions are

$$\mathcal{L}\left\{\left.\frac{\partial\theta_f(x,t)}{\partial x}\right|_{x=0}\right\} = \mathcal{L}\{0\} \quad \text{or} \quad \left.\frac{d\Phi_f(x,S)}{dx}\right|_{x=0} = 0 \tag{A-8}$$

and

$$\mathcal{L}\left\{\theta_f(L,t)\right\} = \mathcal{L}\left\{\sum_{i=1}^{p} B_i\, e^{\beta_i t}\right\} \quad \text{or} \quad \Phi_f(L,S) = \sum_{i=1}^{p} \frac{B_i}{(S-\beta_i)} . \tag{A-9}$$

Applying equation (A-8) to equation (A-7) shows that B must equal zero. A is determined by evaluating equation (A-9). The solution in the transform domain is shown in equation (A-10).

$$\Phi_f(x,S) = \sum_{i=1}^{p} \frac{B_i \cosh\sqrt{\frac{S}{\alpha}}\,x}{(S-\beta_i)\cosh\sqrt{\frac{S}{\alpha}}\,L} - \sum_{j=1}^{s} \frac{q_{oo}\,\alpha A_j}{k(S-\lambda_j)(S-\alpha\kappa^2)} \left\{ \cosh(\kappa x) - \frac{\cosh(\kappa L)\cosh\sqrt{\frac{S}{\alpha}}\,x}{\cosh\sqrt{\frac{S}{\alpha}}\,L} \right\} \tag{A-10}$$

Transforming equation (A-10) back into the time domain is greatly simplified since no poles of order greater than one occur in the inversion integral. It is interesting to note that it is the occurence of higher order poles which complicates the solution to the multiple region problem. Equation (A-11), shown below, (33) can be used to invert transformed functions of the form $\bar{f}(s) = j(s) / \ell(s)$ if the degree of $\ell(s)$ is at least one greater than $j(s)$ and only poles of order one occur,

$$\mathcal{L}^{-1}\left\{\bar{f}(s)\right\} = \sum_{n=1}^{m} \frac{j(\rho_n)}{\ell'(\rho_n)}\, e^{\rho_n t} \tag{A-11}$$

ρ_n denotes the n simple poles of $\bar{f}(S)$ and $\ell'(\rho_n)$ denotes the value of $\frac{d\ell(S)}{dS}$ evaluated at $S = \rho_n$.

The inversion of equation (A-10) is shown in detail below. First consider the third term of equation (A-10).

$$\mathcal{L}^{-1}\left\{\sum_{j=1}^{s} \frac{q_\infty \alpha \cosh \kappa L\, A_j \cosh\sqrt{\frac{S}{\alpha}}\, x}{k(S-\lambda_j)(S-\alpha\kappa^2)\cosh\sqrt{\frac{S}{\alpha}}\, L}\right\} = \sum_{j=1}^{s} \frac{q_\infty\, \alpha \cosh(\kappa L) A_j}{k} \mathcal{L}^{-1}\{I\} \quad \text{(A-12)}$$

The obvious poles of I are at $S = \lambda_j$ and $S = \alpha\kappa^2$. Additional roots of the denominator exist for $\cosh\sqrt{\frac{S}{\alpha}}\, L = 0$. The roots of $\cosh\sqrt{\frac{S}{\alpha}}\, L$ are obtained by making the transformation $S = -\mathcal{s}$. This is done since $\cosh\sqrt{\frac{S}{\alpha}}\, L$ cannot equal zero for real values of the argument and α and L are always positive, real constants.

$$\cosh\sqrt{\frac{S}{\alpha}}\, L = \cosh i\sqrt{\frac{\mathcal{s}}{\alpha}}\, L = \cos\sqrt{\frac{\mathcal{s}}{\alpha}}\, L = 0 \quad \text{(A-13)}$$

therefore

$$\sqrt{\frac{\mathcal{s}}{\alpha}}\, R_o = \frac{n\pi}{2}, \quad n \text{ odd}$$

$$\mathcal{s}_n = \frac{n^2\pi^2\alpha}{4L^2} \quad \text{(A-14)}$$

$$S_n = -\frac{n^2\pi^2\alpha}{4L^2}$$

Evaluating $\mathcal{L}^{-1}\{I\}$ by means of equation (A-11) at the poles $\rho = \alpha\kappa^2$, $\rho = \lambda_j$ and $\rho = -\frac{n^2\pi^2\alpha}{4L^2}$ yields equation (A-15).

$$\mathcal{L}^{-1}\{I\} = \frac{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\,x\right)e^{\lambda_j t}}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\,L\right)(\lambda_j - \alpha\kappa^2)} + \frac{\cosh(\kappa x)\,e^{\alpha\kappa^2 t}}{\cosh(\kappa L)\,(\alpha\kappa^2 - \lambda_j)}$$

$$+\sum_{n=1,3,5\ldots}^{\infty} \frac{\cos\frac{n\pi x}{2L}\, e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)\left(\frac{2L^2}{n\pi\alpha}\right)\sin\left(\frac{n\pi}{2}\right)} \qquad \text{(A-15)}$$

Considering the second term in equation (A-10), the inversion as obtained through use of equation (A-11) is given in equation (A-16).

$$\mathcal{L}^{-1} \sum_{j=1}^{s} \frac{q_{\infty}\alpha A_j}{k} \frac{1}{(S-\lambda_j)(S-\alpha\kappa^2)} = \sum_{j=1}^{s} \frac{q_{\infty}\alpha A_j}{k}\left(\frac{e^{\lambda_j t}}{(\lambda_j - \alpha\kappa^2)} + \frac{e^{\alpha\kappa^2 t}}{(\alpha\kappa^2 - \lambda_j)}\right) \qquad \text{(A-16)}$$

In a similar manner the first term of equation (A-10), the inversion, again using equation (A-11), is given in equation (A-17).

$$\mathcal{L}^{-1}\left\{\sum_{i=1}^{p} \frac{C_i \cosh\sqrt{\frac{s}{\alpha}}\,x}{(S-\beta_i)\cosh\sqrt{\frac{s}{\alpha}}\,L}\right\} = \sum_{i=1}^{p} \frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\,x\right)e^{\beta_i t}}{\cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\,L\right)}$$

$$+\sum_{i=1}^{p}\sum_{n=1,3,5,\cdots}^{\infty} \frac{B_i \cos\frac{n\pi x}{2L}\, e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{\left(-\frac{n^2\pi^2\alpha}{4L^2} - \beta_i\right)\frac{L^2}{n\pi\alpha}\left(\sin\frac{n\pi}{2}\right)} \qquad \text{(A-17)}$$

The temperature distribution in the fuel plate, $\underline{\theta_f(x,t)}$ is obtained by substituting equation (A-15) in equation (A-12) and adding equations (A-12), (A-16) and (A-17). The result is

$$\theta_f(x,t) = \sum_{i=1}^{p} \frac{B_i \cosh\sqrt{\frac{\beta_i}{\alpha}}\,x}{\cosh\sqrt{\frac{\beta_i}{\alpha}}\,L}\, e^{\beta_i t} - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\frac{n\pi x}{2L}\, e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{\left(\frac{L^2}{\alpha n\pi}\right)\sin\left(\frac{n\pi}{2}\right)}$$

$$X \left\{ \sum_{i=1}^{p} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} + \sum_{j=1}^{s} \frac{q_{oo}\,\alpha\, A_j \cosh \kappa L}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)} \right\} \qquad \text{(A-18)}$$

$$- \sum_{j=1}^{s} \frac{A_j\, q_{oo}\, \alpha\, e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{ \cosh(\kappa x) - \frac{\cosh(\kappa L)\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\,x\right)}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\,L\right)} \right\}$$

As seen by comparing (A-1) and (A-2) the differential equations to be solved in the fuel and moderator are almost the same, the only difference being in the heat generation term. Thus, the total differential equation for the moderator in the Laplace transform domain after having applied the zero initial temperature condition is given in equation (A-19)

$$\nabla^2 \Phi_m(x,s) - \frac{S}{\alpha}\Phi_m(x,S) = - \sum_{j=1}^{s} \frac{F\, A_j}{k\,(S-\lambda_j)} \qquad \text{(A-19)}$$

The homogeneous solutions are the same as before and particular solutions are easily obtained, as before, from the method of undetermined coefficients. Therefore, the solutions to equation (A-19) in the transform domain are

$$\Phi_m(x,S) = C \cosh\sqrt{\frac{S}{\alpha}}\,X, + D \sinh\sqrt{\frac{S}{\alpha}}\,X, + \sum_{j=1}^{s} \frac{F\,\alpha\, A_j}{k\, S(S-\lambda_j)} \qquad \text{(A-20)}$$

The transform boundary conditions for the moderator are same as those for the fuel if the origin in the moderator is taken at the outside of the unit cell, i.e.,

$$\Phi_m(L,S) = \sum_{i=1}^{p} \frac{B_i}{S-\beta_i} \qquad \text{and} \qquad \left.\frac{d\Phi\,(x_1\,,\ S)}{d\,x_1}\right|_{x_1=0} = 0\ . \tag{A-21}$$

Thus $\underline{D}$ equals zero and $\underline{A}$ takes the same form as for the solution in the fuel. The complete solution in the transform domain is given in equation (A-22).

$$\Phi_m(x,S) = \sum_{i=1}^{p} \frac{B_i \cosh\sqrt{\frac{S}{\alpha}}\,x_1}{(S-\beta_i)\cosh\sqrt{\frac{S}{\alpha}}\,L_1} + \sum_{j=1}^{s} \frac{\alpha\,F\,A_j}{k\,S\,(S-\lambda_j)}$$

$$\left[\,1 - \frac{\cosh\sqrt{\frac{S}{\alpha}}\,x_1}{\cosh\sqrt{\frac{S}{\alpha}}\,L_1}\,\right]. \tag{A-22}$$

The inversion of this solution is easily accomplished by the same method used for the fuel region. The solution for the temperature is

$$\theta_m(x,t) = \sum_{i=1}^{p} \frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\,x_1\right)e^{\beta_i t}}{\cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\,L_1\right)} - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\frac{n\pi x_1}{2L_1}\cdot e^{\frac{-n^2\pi^2\alpha}{4L_1^2}t}}{\left(\frac{L_1^2}{n\pi\alpha}\right)\sin\left(\frac{n\pi}{2}\right)}$$

$$\times \left\{\sum_{i=1}^{p} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L_1^2} + \beta_i} + \sum_{j=1}^{s} \frac{\alpha\,F\,A_j}{\frac{n^2\pi^2\alpha}{4L_1^2}\left(\frac{n^2\pi^2\alpha}{4L_1^2} + \lambda_j\right)}\right\} \tag{A-23}$$

$$+ \sum_{j=1}^{s} \frac{\alpha F A_j e^{\lambda_j t}}{k\lambda_j} \left\{1 - \frac{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,x_1}{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,L_1}\right\}.$$

This solution could have been obtained from the solution in the fuel by setting $\kappa = 0$ and $q_{oo} = F$.

The equivalent solution in cylindrical geometry (r dependence only) for the exponential boundary condition is obtained in the same general manner, however, several important differences do occur. The equations governing the temperature in the fuel and moderator are

$$\nabla^2\theta_f(r,t) + \sum_{j=1}^{s} \frac{q_{oo} I_o(\kappa r) A_j e^{\lambda_j t}}{k} = \frac{1}{\alpha} \frac{\partial\theta_f(r,t)}{\partial t} \qquad \text{(A-24)}$$

and

$$\nabla^2\theta_m(r,t) + \sum_{j=1}^{s} \frac{F A_j e^{\lambda_j t}}{k} = \frac{1}{\alpha} \frac{\partial\theta_m(x,t)}{\partial t} \qquad \text{(A-25)}$$

Considering the fuel region first, transforming with respect to time and applying the zero initial condition yields

$$\nabla^2\Phi(r,S) - \frac{S}{\alpha}\Phi(r,S) = -\sum_{j=1}^{s} \frac{q_{oo} A_j I_o(\kappa r)}{k(S-\lambda_j)} \qquad \text{(A-26)}$$

The spatial operator for this case is

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \qquad \text{(A-27)}$$

The particular solution, again by the method of undetermined coefficients, is

$$\Phi_p(r,S) = \frac{q_{oo}\,\alpha A_j I_o(\kappa r)}{k(S-\lambda_j)(S-\alpha\kappa^2)} \qquad \text{(A-28)}$$

The homogeneous solution, noting the fact that $\Phi(o,S)$ if finite is

$$\Phi_h(r,s) = A\, I_o \sqrt{\frac{S}{\alpha}}\, r \qquad \text{(A-29)}$$

The final constant $\underline{A}$ is evaluated by use of the surface temperature boundary condition and the solution in the transform domain is

$$\Phi_f(r,S) = \sum_{i=1}^{p} \frac{B_i}{S-\beta_i} \frac{I_o\sqrt{\frac{S}{\alpha}}\,r}{I_o\sqrt{\frac{S}{\alpha}}\,R_o} + \sum_{j=1}^{s} \frac{q_{\infty}\,\alpha\,A_j}{k(S-\lambda_j)(S-\alpha\kappa^2)} \left(I_o(\kappa r) - \frac{I_o(\kappa R_o)\sqrt{\frac{S}{\alpha}}\,r}{I_o\sqrt{\frac{S}{\alpha}}\,R_o} \right) . \tag{A-30}$$

Inverting this expression is entirely analogous to the inversion of the equivalent expression, equation (A-9) of this appendix. The analogy carries even to the point that in determining the zeros of $I_o\sqrt{\frac{S}{\alpha}}\,R_o$ as in determining the zeros of Cosh $\sqrt{\frac{S}{\alpha}}$ L imaginary values of S yield an infinite set of zeros. In this case the zeros are

$$I_o\sqrt{\frac{S}{\alpha}}\,R_o = I_o\sqrt{\frac{-s}{\alpha}}\,R_o = I_o\,i\sqrt{\frac{s}{\alpha}}\,R_o = J_o\sqrt{\frac{s}{\alpha}}\,R_o = 0 \tag{A-31}$$

therefore,

$\sqrt{\frac{s}{\alpha}}\,R_o = \omega_n$, where $\omega_n = 2.4048,\ 5.5201$, etc. (the zeros of $J(\omega_n)=0$)

$$\frac{s}{\alpha} R_o^2 = \omega_n^2 ,$$

$$s_n = \frac{\omega_n^2 \alpha}{R_o^2} \tag{A-32}$$

and

$$S_n = \frac{\omega_n^2\,\alpha}{R_o^2}$$

The details of the remainder of the inversion are not included. The result is

$$\theta_f(x,t) = \sum_{i=1}^{p} \frac{B_i\, I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right) e^{\beta_i t}}{I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, R\right)} + \sum_{n=1,2,3,\cdots}^{\infty} \frac{J_o\left(\frac{\omega_n r}{R}\right) e^{-\frac{\omega_n^2 \alpha}{R^2} t}}{\frac{R^2}{2\omega_n \alpha}\, J\,(\omega_n)}$$

$$\times \left\{ \sum_{i=1}^{p} \frac{B_i}{\frac{\omega_n^2 \alpha}{R^2} + \beta_i} + \sum_{j=1}^{s} \frac{q_\infty\, \alpha\, A_j\, I_o(\kappa R_o)}{k\left(\frac{\omega_n^2 \alpha}{R^2} + \lambda_j\right)\left(\frac{\omega_n^2 \alpha}{R^2} + \alpha\kappa^2\right)} \right\} \qquad \text{(A-33)}$$

$$- \sum_{j=1}^{s} \frac{A_j\, q_\infty\, \alpha\, e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j t)} \left\{ [I_o(\kappa r)] - \frac{[I_o(\kappa R)\, I_o\sqrt{\frac{\lambda_j}{\alpha}}\, r]}{I_o\sqrt{\frac{\lambda_j}{\alpha}}\, R} \right\},$$

where the ω_n's are the roots of the equation, $J_o(X) = 0$.

The solution for the cylindrical geometry in the moderator is complicated only by the fact that the symmetry condition cannot be located at the coordinate $r = 0$. Thus both terms $I_o\sqrt{\frac{S}{\alpha}}\, r$ and $K_o\sqrt{\frac{S}{\alpha}}\, r$ of the homogeneous solution must be retained. The details of this solution are not included. The result is

$$\theta_m(x,t) = \sum_{i=1}^{p} B_i \left\{ \frac{K_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right) + I_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) K_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, r\right)}{\left(K_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) I_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, R\right) + I_1\left(\sqrt{\frac{\beta_i}{\alpha}}\, R_1\right) K_o\left(\sqrt{\frac{\beta_i}{\alpha}}\, R\right)\right)} \right\} e^{\beta_i t}$$

$$+\sum_{n=1}^{\infty} \frac{2\sqrt{\rho_n\alpha}\,\left[K_1\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right) I_0\left(\sqrt{\frac{\rho_n}{\alpha}}r\right) + I_1\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right) K_0\left(\sqrt{\frac{\rho_n}{\alpha}}r\right)\right] e^{\rho_n t}}{R\left[K_1\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right) I_1\left(\sqrt{\frac{\rho_n}{\alpha}}R\right) - K_1\left(\sqrt{\frac{\rho_n}{\alpha}}R\right) I_1\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right) - I_0\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right) K_0\left(\sqrt{\frac{\rho_n}{\alpha}}R\right) + I_0\left(\sqrt{\frac{\rho_n}{\alpha}}R\right) K_0\left(\sqrt{\frac{\rho_n}{\alpha}}R_1\right)\right]}$$

$$\times\left\{\sum_{i=1}^{p} \frac{B_i}{\rho_n-\beta_i} - \sum_{j=1}^{s} \frac{\alpha F A_j}{k\,\rho_n(\rho_n-\lambda_j)}\right\} + \sum_{j=1}^{s} \frac{\alpha F A_j e^{\lambda_j t}}{k} \qquad \text{(A-34)}$$

$$\times\left\{\frac{K_1\left(\sqrt{\frac{\lambda_j}{\alpha}}R_1\right) I_0\left(\sqrt{\frac{\lambda_j}{\alpha}}r\right) + I_1\left(\sqrt{\frac{\lambda_j}{\alpha}}R_1\right) K_0\left(\sqrt{\frac{\lambda_j}{\alpha}}r\right)}{K_1\left(\sqrt{\frac{\lambda_j}{\alpha}}R_1\right) I_0\left(\sqrt{\frac{\lambda_j}{\alpha}}R\right) I_1\left(\sqrt{\frac{\lambda_j}{\alpha}}R_1\right) K_0\left(\sqrt{\frac{\lambda_j}{\alpha}}R\right)}\right\},$$

where the ρ_n's are the roots of the equation,

$$K_1\left(\sqrt{\frac{S}{\alpha}}R_1\right) I_0\left(\sqrt{\frac{S}{\alpha}}R\right) + I_1\left(\sqrt{\frac{S}{\alpha}}R_1\right) K_0\left(\sqrt{\frac{S}{\alpha}}R\right) = 0$$

The next solution considered again uses an exponential fit for the heat generation rate, however, the boundary condition was that the first derivative with respect to x evaluated at the outside of the plate or effectively the heat flow out of the plate could be expressed as a sum of exponentials. The general solution with the exception of the evaluation of the final constant <u>A</u> is exactly the same as the first derivation in this appendix. Including the one undetermined coefficient the solution is

$$\Phi_f(x,S) = A\cosh\sqrt{\frac{S}{\alpha}}\,x + \sum_{j=1}^{s} \frac{q_{oo}\,\alpha\,A_j\cosh\kappa x}{k(S-\lambda_j)\,(S-\alpha\kappa^2)}\,. \qquad \text{(A-35)}$$

Evaluation of A through use of the boundary condition leads to

$$\Phi_f(x,S) = \sum_{i=1}^{p'} \frac{B_i \cosh\left(\sqrt{\frac{S}{\alpha}}\,x\right)}{(S-\beta_i)\sqrt{\frac{S}{\alpha}}\,\sinh\left(\sqrt{\frac{S}{\alpha}}\,L\right)} + \sum_{j=1}^{s'} \frac{q_{oo}\alpha A_j}{k(S-\lambda_j)(S-\alpha\kappa^2)}$$

$$\left(\cosh \kappa x - \frac{\cosh(\kappa L)\cosh\left(\sqrt{\frac{S}{\alpha}}\,x\right)}{\cosh\left(\sqrt{\frac{S}{\alpha}}\,L\right)} \right) \quad . \tag{A-36}$$

The details of this inversion are not included. The result is

$$\theta_f(x,t) = \sum_{i=1}^{p'} \left(\frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\,x\right)e^{\beta_i t}}{\sqrt{\frac{\beta_i}{\alpha}}\,\sinh\left(\sqrt{\frac{\beta_i}{\alpha}}\,L\right)} - \frac{B_i\alpha}{\beta_i L}\right) - \sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi x}{2L}\right) e^{-\frac{n^2\pi^2 x}{4L^2}}}{\frac{L}{\alpha}\cos n\pi}$$

$$X \left\{ \sum_{i=1}^{p'} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} + \sum_{j=1}^{s'} \frac{q_{\infty}\,\alpha A_j\,\kappa \sinh \kappa L}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)} \right\} \tag{A-37}$$

$$-\sum_{j=1}^{s'} \frac{q_{\infty}\,\alpha A_j\,e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left[\cosh \kappa x - \frac{\kappa \sinh(\lambda L)\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\,x\right)}{\sqrt{\frac{\lambda_j}{\alpha}}\,\sinh\left(\sqrt{\frac{\lambda_j}{\alpha}}\,L\right)}\right] - \frac{q_{\infty}\,\alpha A_j \sinh \kappa L}{k\,\lambda_j\,\kappa L} \; .$$

The time dependent temperature distribution in the moderator for the comparable boundary conditions is obtained by setting κ equal to zero and q_{∞} equal to F in equation (A-37). The result is

$$\theta_m(x,t) = \sum_{i=1}^{p'} \left(\frac{B_i \cosh\left(\sqrt{\frac{\beta_i}{\alpha}}\,x\right)e^{\beta_i t}}{\sqrt{\frac{\beta_i}{\alpha}}\,\sinh\left(\sqrt{\frac{\beta_i}{\alpha}}\,L\right)} - \frac{B_i\alpha}{\beta_i L}\right)$$

$$- \sum_{n=1}^{\infty} \frac{\cos \frac{n\pi x}{2L} \; e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{\left(\frac{L}{\alpha}\right) \cos n\pi} \left\{ \sum_{i=1}^{p'} \frac{B_i}{\frac{n^2\pi^2\alpha}{4L^2} + \beta_i} \right\}$$

$$+ \sum_{j=1}^{s'} \frac{\alpha F A_j}{k(\lambda_j)} \left(e^{\lambda_j t} - 1 \right). \tag{A-38}$$

The final solution considered again used a sum of exponentials to represent the time dependence of the heat generation rate, however, the surface temperature boundary condition was approximated by an even Fourier series, $\sum_{i=1}^{p} B_i \cos \beta_i t$. With the exception of the steady state term resulting from the surface temperature boundary condition the derivation follows exactly the first derivation of this appendix. The steady state term is handled most easily in a slightly different manner as shown below. The general solution in the transform domain is

$$\Phi_f(x,S) = \sum_{i=1}^{p} \frac{B_i S \cosh\left(\sqrt{\frac{S}{\alpha}}\, x\right)}{(S^2+\beta_i^2) \cosh\left(\sqrt{\frac{S}{\alpha}}\, L\right)} + \sum_{j=1}^{s} \frac{q_{\infty}\, \alpha\, A_j}{k(S-\lambda_j)\,(S-\alpha\kappa^2)}$$

$$\left[\cosh \kappa x - \frac{\cosh(\kappa L) \cosh\left(\sqrt{\frac{S}{\alpha}}\, x\right)}{\cosh\left(\sqrt{\frac{S}{\alpha}}\, L\right)}\right] \tag{A-39}$$

The first term presents the only change and there only for the poles at $S = \pm j\beta_i$. The terms generated from the inversion integral by these two poles are

$$\rho_1 \Big|_{S=j\beta_i} = \frac{B_i\, j\, \beta_i \cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\, x\right)}{2\, j\, \beta_i \cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\, L\right)}\, e^{j\beta_i t} \tag{A-40}$$

and

$$\rho_2 \Bigg|_{S=-j\beta_i} = \frac{B_i\ (-j\beta_i)\ \cosh\left(\sqrt{\frac{-j\beta_i}{\alpha}}\ x\right)}{2\ (-j\beta_i)\ \cosh\left(\sqrt{\frac{-j\beta_i}{\alpha}}\ L\right)}\ e^{-j\beta_i t} \tag{A-41}$$

These terms are most easily handled by recognizing the fact that the sum $\rho_1 + \rho_2$ is the sum of a function and its conjugate. That is

$\rho_1 + \rho_2 = f(z) + f(\bar{z}) = f(z) + \overline{f(z)}$ since z is a pure imaginary number.

Therefore

$$\rho_1 + \rho_2 = 2\ \mathrm{Re}\left\{ f(z) \right\} = 2\ \mathrm{Re}\left\{ \rho_1 \right\}$$

$$= 2\ \mathrm{Re}\left\{ \frac{B_i\ \cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\ x\right)}{2\ \cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\ L\right)}\ e^{j\beta_i t} \right\} \tag{A-42}$$

$$= B_i\ R_e \left\{ \frac{\cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\ x\right)}{\cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\ L\right)}\ e^{j\beta_i t} \right\}$$

$$= B_i\ R_e \left| \frac{\cosh\left(\sqrt{\frac{j\beta_i}{\alpha}}\ x\right)}{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\ L} \right| e^{j\left(\beta_i t + \arg\left\{ \frac{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\ x}{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\ L} \right\}\right)}$$

$$= B_i\ Z_i^{\frac{1}{2}}(x)\ \cos\left[\beta_i t + \varphi_i(x)\right],$$

where

$$Z_i(x) = \left| \frac{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\ x}{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\ L} \right|^2 = \left| \frac{\cosh\left(\sqrt{\frac{\beta_i}{2\alpha}}\ x\right)\cos\left(\sqrt{\frac{\beta_i}{2\alpha}}\ x\right) + j\ \sin\left(\sqrt{\frac{\beta_i}{2\alpha}}\ x\right)\sinh\left(\sqrt{\frac{\beta_i}{2\alpha}}\ x\right)}{\cosh\left(\sqrt{\frac{\beta_i}{2\alpha}}\ L\right)\cos\left(\sqrt{\frac{\beta_i}{2\alpha}}\ L\right) + j\ \sin\left(\sqrt{\frac{\beta_i}{2\alpha}}\ L\right)\sinh\left(\sqrt{\frac{\beta_i}{2\alpha}}\ L\right)} \right|^2$$

$$= \left\{ \frac{\cos^2(\sqrt{\frac{\beta_i}{2\alpha}}\,x)\cosh^2(\sqrt{\frac{\beta_i}{2\alpha}}\,x) + \sin^2(\sqrt{\frac{\beta_i}{2\alpha}}\,x)\sinh^2(\sqrt{\frac{\beta_i}{2\alpha}}\,x)}{\cos^2(\sqrt{\frac{\beta_i}{2\alpha}}\,L)\cosh^2(\sqrt{\frac{\beta_i}{2\alpha}}\,L) + \sin^2(\sqrt{\frac{\beta_i}{2\alpha}}\,L)\sinh^2(\sqrt{\frac{\beta_i}{2\alpha}}\,L)} \right\} \tag{A-43}$$

and

$$\varphi_i(x) = \arg\left\{\frac{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\,x}{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\,L}\right\} = \arg\left\{\cosh\sqrt{\frac{j\beta_i}{\alpha}}\,x\right\} - \left\{\arg\ \cosh\sqrt{\frac{j\beta_i}{\alpha}}\,L\right\}$$

$$= \tan^{-1}\left(\frac{\sin\sqrt{\frac{\beta_i}{2\alpha}}\,x\ \sinh(\sqrt{\frac{\beta_i}{2\alpha}}\,x)}{\cos\sqrt{\frac{\beta_i}{2\alpha}}\,x\ \cosh(\sqrt{\frac{\beta_i}{2\alpha}}\,x)}\right) - \tan^{-1}\left(\frac{\sin(\sqrt{\frac{\beta_i}{2\alpha}}\,L)\sinh(\sqrt{\frac{\beta_i}{2\alpha}}\,L)}{\cos(\sqrt{\frac{\beta_i}{2\alpha}}\,L)\cosh(\sqrt{\frac{\beta_i}{2\alpha}}\,L)}\right). \tag{A-44}$$

The resultant time dependent temperature distributions are

$$\theta_f(x,t) = \sum_{i=1}^{p} B_i\, Z_i^{\frac{1}{2}}\ \mathrm{Cos}\,(\beta_i t + \varphi_i) - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\frac{n\pi x}{2L}\ e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{(L^2/n\pi\alpha)\ \sin\frac{n\pi}{2}}$$

$$\times\left\{\sum_{i=1}^{p} \frac{B_i\ (\frac{n^2\pi^2\alpha}{4L^2})}{(\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2)} + \sum_{j=1}^{s} \frac{q_{oo}\ \alpha\ A_j\ \cosh\kappa L}{k(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2)(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j)}\right\}. \tag{A-45}$$

$$- \sum_{j=1}^{s} \frac{q_{oo}\alpha\ A_j\ e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{\cosh\kappa x - \frac{\cosh(\kappa L)\cosh(\sqrt{\frac{\lambda_j}{\alpha}}\,x)}{\cosh(\sqrt{\frac{\lambda_j}{\alpha}}\,L)}\right\}$$

and

$$\theta_m(x,t) = \sum_{i=1}^{p} B_i Z_i^{\frac{1}{2}} \cos(\beta_i t + \varphi_i) + \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\frac{n\pi x}{2L} e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{(L^2/n\pi\alpha)\sin\frac{n\pi}{2}}$$

$$\times\left\{\sum_{i=1}^{p} \frac{B_i\left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2} - \sum_{j=1}^{p} \frac{\alpha F A_j}{k\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)}\right\} \tag{A-46}$$

$$+\sum_{j=1}^{s} \frac{\alpha F A_j e^{\lambda_j t}}{k(\lambda_j)} \left\{1 - \frac{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,x}{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,L}\right\}$$

in the fuel and moderator, respectively.

APPENDIX B

Description and Explanation of the IBM-650 Computer Program Used for Fitting Empirically Experimental Data with the Sum of Several Terms of Exponential Form

The computer code was written to fit an analytical function of the form of the sum of exponentials to the experimentally determined power traces during a transient burst. The program was written in SOAP II and floating point form. The object program is listed and the logic diagram is shown in this appendix.

The criteria that the machine inspected was that the sum of the squares of the residuals between the experimental data and the calculated values should be made as small as possible. Each of the fitting parameters was varied in turn by a specified increment, holding all other parameters constant, until such a time that a specified increment could make no further reduction in the sum of the squares of the residuals. This parameter was then stored as the best available estimate of the particular empirical parameter. When none of the parameters could be varied by the specified increment to give a smaller sum of the squares of the residuals, the increments were refined and the trial and error process was repeated with the refined increments. This procedure continued until the increments were less than a specified precision.

The data were fit empirically with a function of the form

$$P_i = \sum_{j=1}^{s} A_j \, e^{\lambda_j t_i} \qquad s \leq 10 \qquad \text{(B-1)}$$

As stated above, the best fit criterion was that

$$\text{ERROR} = \sum_{i=1}^{c} \frac{1}{\omega_i^2} \sum_{j=1}^{s} \left(A_j \, e^{\lambda_j t_i} - \Phi_i \right)^2 \qquad \text{(B-2)}$$

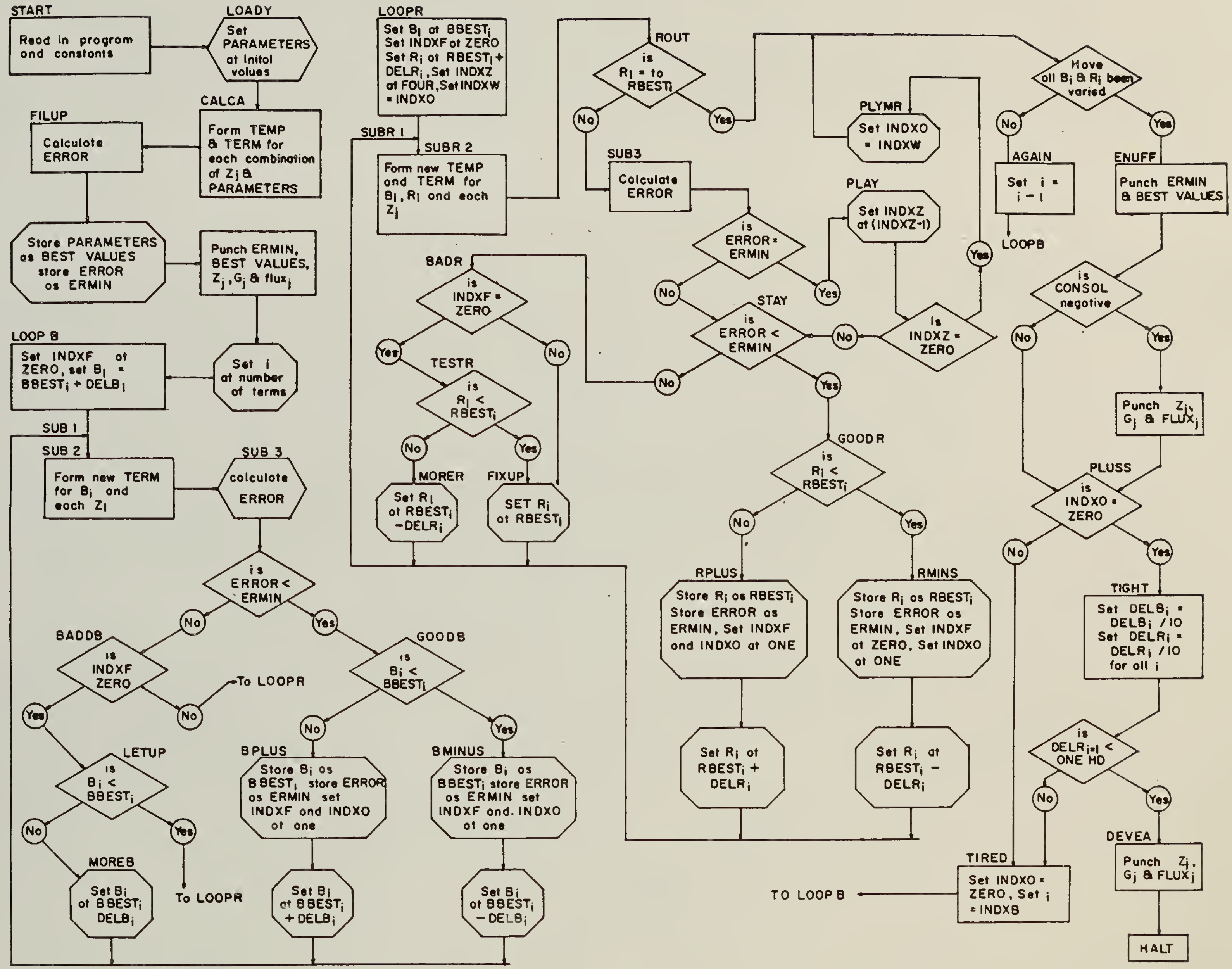
be a minimum.

The program required, in addition to the experimental data and their respective times, initial estimates for a specific number of parameters. The program had a capacity for up to 20 data points and 10 terms in the summation of equation (B-1). These input data were read into the machine along with the program deck on one-word load cards. Each one-word load card contained a particular constant or an initial value and its specified storage location. Table B-1 lists the various input data needed for this program.

Table B-1. Input data required for use of the IBM-650 program which fit empirically experimental data with several terms of Exponential form.

Symbol	Explanation	Storage Location
ZERD	0.00	0073
FPONE	1.00	0129
FOUR	4.00	0185
TEN	10.00	0158
HNDRD	100.00	0090
ONE	Index Number 1 (0000000001)	0392
EIGHT	Index Number 8 (0000000008)	0024
INDXB	Number of Exponential Terms (00000000xx)	0412
INDXA	Number of Data Points (00000000xx)	0062
ONEHD	Precision	0168
$DELB_j$	Initial increments of the Amplitude, B_j	(1800 + i)
$DELR_j$	Initial increments of R_j	(1810 + j)
$BINIT_j$	Initial estimate of B_j	(1300 + j)
$RINIT_j$	Initial estimate of R_j	(1310 + j)
Z_i	Time of ith data point	(1200 + i)
$FLUX_i$	Experimental data at ith point	(1200 + i)
W_i	Weighting function at ith point	(1750 + i)

The machine yielded an answer card having a capacity of 8 words, a word being ten digit numbers and a sign. For the first answer the machine punched out the initial estimates of the fitting parameters on as many cards as was necessary to accommodate them. $\underline{A}$ and $\underline{\lambda}$ for the first term were stored in word locations 1 and 2, respectively. After a card was filled to its 8 word capacity, it was punched and a new card began to fill. This procedure continued until all of the fitting parameters had been punched out. Then a separate card was punched giving in the word 8 location, the value of the sum of the weighted of the residuals between the experimental data and the calculated values. The machine then punched out values of the time, residual and correct values at the last data point in word locations 1, 2 and 3, respectively and the next to last data point in word locations 5, 6 and 7, respectively. The same information for two previous data points was punched out on a second card in the same format and this procedure was continued until the position residual and correct value was punched out for each data point. Subsequent improvements in the parameters and the weighted sum of squares of the residuals were printed out after each cycle of trying to vary each parameter. The positions, residuals and correct values at each data point were obtained at this time if the console instruction was negative. When the fitting parameters could not be further improved with the most refined increment specified, the punching of the best fit parameters, the sum of the weighted square of the residuals, the position, residual and correct values took place according to the procedure described above.



LOGIC DIAGRAM-APPENDIX B

OBJECT PROGRAM-APPENDIX 8

```
          8LR  1200     1830            1   0000   00 0000 0000
          8LR  1951     1960            2   0000   00 0000 0000
          8LR  1977     1985            3   0000   00 0000 0000
          8YN  8TART    1999            4   0000   00 0000 0000
          8YN  Z        1800            5   0000   00 0000 0000
          8YN  FLUX     1220            6   0000   00 0000 0000
          8YN  G        1240            7   0000   00 0000 0000
          8YN  8        1260            8   0000   00 0000 0000
          8YN  8BEST    1270            9   0000   00 0000 0000
          8YN  R        1280           10   0000   00 0000 0000
          8YN  RBEST    1290           11   0000   00 0000 0000
          8YN  8INIT    1300           12   0000   00 0000 0000
          8YN  RINIT    1310           13   0000   00 0000 0000
          8YN  DEL8     1800           14   0000   00 0000 0000
          8YN  DELR     1810           15   0000   00 0000 0000
          8YN  TEMP     1320           16   0000   00 0000 0000
          8YN  TERM     1520           17   0000   00 0000 0000
          8YN  C        1720           18   0000   00 0000 0000
          8YN  FRAC     1730           19   0000   00 0000 0000
          8YN  QUOT     1740           20   0000   00 0000 0000
          8YN  W        1750           21   0000   00 0000 0000
PRNTM     8TO  EXXXT                   22   0000   24 0003 0006
          LDO           EDDCL          23   0006   69 0009 0012
          LOD  INOXA                   24   0009   69 0062 0015
          RAA  8001     RE88T          25   0015   80 8001 0021
RE88T     LOO  EIGHT                   26   0021   69 0024 0027
          R88  8001     8TUUF          27   0027   83 8001 0033
8TUUF     LOO  Z      A                28   0033   69 3200 0053
          8TO  1985   8                29   0053   24 5985 0038
          LOO  G      A                30   0038   69 3240 0043
          8TD  1986   8                31   0043   24 5986 0039
          LDD  FLUX   A                32   0039   69 3220 0023
          8TO  1987   8                33   0023   24 5987 0040
          8XA  0001                    34   0040   51 0001 0046
          AX8  0004                    35   0046   52 0004 0002
          NZA  MDRRG    PONNM          36   0002   40 0005 0056
PONNM     PCH  1977     EXXXT          37   0056   71 1977 0003
MDRRG     NZB  8TUUF    CARRO          38   0005   42 0033 0059
CARRU     PCH  1977                    39   0059   71 1977 0077
          LOO  RE88T    EDDCL          40   0077   69 0021 0012
EOOEA     8TD  AAA1            E       41   0050   24 0103 0106
          8TL  AAA14                   42   0106   20 0011 0014
          RAU  AAA16           Z       43   0014   60 0017 0071
          FAM  AAA14                   44   0071   37 0011 0037
          8TU  AAA2                    45   0037   21 0042 0045
          LOD  AAA3            O       46   0045   69 0040 0001
          8TD  AAA4     AAA8   C       47   0001   24 0004 0007
AAA8      RAU  AAA2                    48   0007   60 0042 0047
          F88  AAA5            F       49   0047   33 0100 0127
          8MI  AAA6                    50   0127   46 0030 0031
          8TU  AAA2                    51   0031   21 0042 0095
          RAU  AAA4                    52   0095   60 0004 0109
          FMP  AAA7            E       53   0109   39 0112 0162
          8TU  AAA4     AAA8           54   0162   21 0004 0007
AAA6      RAU  AAA2                    55   0030   60 0042 0097
          F88  AAA3            O       56   0097   33 0048 0025
          8MI  AAA28                   57   0025   46 0028 0029
          8TU  AAA2                    58   0029   21 0042 0145
          RAU  AAA4                    59   0145   60 0004 0159
          FMP  AAA9            E       60   0159   39 0212 0262
          8TU  AAA4     AAA6           61   0262   21 0004 0030
AAA28     RAU  AAA2                    62   0028   60 0042 0147
          F88  AAA10           P       63   0147   33 0150 0177
          8MI  AAA11                   64   0177   46 0080 0081
          8TU  AAA2                    65   0081   21 0042 0195
          RAU  AAA4                    66   0195   60 0004 0209
          FMP  AAA12           E       67   0209   39 0312 0362
          8TU  AAA4     AAA28          68   0362   21 0004 0028
AAA11     RAU  AAA2                    69   0080   60 0042 0197
          LOD           AAA17          70   0197   69 0200 0153
          FMP  AAA4                    71   0200   39 0004 0054
          8TU  AAA13                   72   0054   21 0008 0061
          RAU  AAA14                   73   0061   60 0011 0065
          8MI  AAA15                   74   0065   46 0018 0019
          RAL  AAA13    AAA1           75   0019   65 0008 0103
AAA15     RAU  AAA3                    76   0018   60 0048 0203
          FOV  AAA13                   77   0203   34 0008 0058
          RAL  8003     AAA1           78   0058   65 8003 0103
AAA17     8TO  AAA18           E       79   0153   24 0156 0259
          RAU  AAA3            D       80   0259   60 0048 0253
          FAD  AAA2                    81   0253   32 0042 0069
          8TU  AAA19                   82   0069   21 0074 0227
          LOO  AAA27           T       83   0227   69 0130 0083
          8TO  AAA20           N       84   0083   24 0036 0089
          8TO  AAA21           O       85   0089   24 0092 0245
          RAU  AAA2                    86   0245   60 0042 0247
          FMP  AAA2                    87   0247   39 0042 0142
          8TU  AAA23    AAA22  N       88   0142   21 0096 0049
AAA22     FOV  AAA21                   89   0049   34 0092 0192
          8TU  AAA24           N       90   0192   21 0146 0099
          FAO  AAA19                   91   0099   32 0074 0051
          8TU  AAA19           T       92   0051   21 0074 0277
          RAU  AAA24                   93   0277   60 0146 0101
          FDV  AAA19                   94   0101   34 0074 0124
          F88  AAA25           C       95   0124   33 0327 0303
          8MI           AAA26          96   0303   46 0206 0057
          RAU  AAA19    AAA18          97   0206   60 0074 0156
AAA26     RAU  AAA20                   98   0057   60 0036 0041
          FAD  AAA3                    99   0041   32 0048 0075
          8TU  AAA20           N      100   0075   21 0036 0139
          FMP  AAA21                  101   0139   39 0092 0242
          8TU  AAA21           N      102   0242   21 0092 0295
          RAU  AAA23                  103   0295   60 0096 0151
          FMP  AAA2                   104   0151   39 0042 0292
          8TU  AAA23    AAA22         105   0292   21 0096 0049
AAA3       10  0000     0051   O      106   0048   10 0000 0051
AAA5       50  0000     0051   F      107   0100   50 0000 0051
AAA7       14  8410     0053   E      108   0112   14 8410 0053
AAA9       27  1830     0051   E      109   0212   27 1830 0051
AAA10      20  0000     0050   P      110   0150   20 0000 0050
AAA12      12  2140     0051   E      111   0312   12 2140 0051
AAA16      00  0000     0000   Z      112   0017   00 0000 0000
AAA25      70  0000     0047   C      113   0327   70 0000 0047
AAA27      20  0000     0051   T      114   0130   20 0000 0051
EOOCL     8TD  ZZZ1                   115   0012   24 0115 0068
```

```
         LOD  ZZZ10               116   0068   69  0121  0174
         STO   1977               117   0174   24  1977  0180
         STO   1978               118   0180   24  1978  0131
         STO   1979               119   0131   24  1979  0032
         STO   1980               120   0032   24  1980  0133
         STO   1981               121   0133   24  1981  0034
         STO   1982               122   0034   24  1982  0035
         STO   1983               123   0035   24  1983  0086
         STO   1984     ZZZ1      124   0086   24  1984  0115
ZZZ10     00   0000      0000     125   0121   00  0000  0000
TEMPP    STO  EGSIT               126   0250   24  0353  0256
         RAU  Z      A            127   0256   60  3200  0055
         FMP  R      B            128   0055   39  5280  0230
         STU  RZEE                129   0230   21  0084  0087
         FAD  HNDRD               130   0087   32  0090  0067
         BMI  STO       GO        131   0067   46  0020  0171
STO      LOD  ZERO                132   0020   69  0073  0026
         STO  TEMP   C  EGBIT     133   0026   24  7320  0353
GO       RAL  RZEE                134   0171   65  0084  0189
         LOD            EOOEA     135   0189   69  0342  0050
         STL  TEMP   C  EGBIT     136   0342   20  7320  0353
TURMM    STO  EXXIT               137   0300   24  0403  0306
         RAU  TEMP   C            138   0306   60  7320  0125
         FMP  B      B            139   0125   39  5260  0010
         STU  TERM   C  EXXIT     140   0010   21  7520  0403
ERR      STO  EXIT                141   0350   24  0453  0356
         LOD  ZERO                142   0356   69  0073  0076
         RAC   8001               143   0076   88  8001  0082
         STO  ERROR               144   0082   24  0085  0088
         LOD  INDXA               145   0088   69  0062  0165
         RAA   8001     READY     146   0165   80  8001  0221
READY    LOD  INDXC               147   0221   69  0224  0377
         AXC   8001               148   0377   58  8001  0183
         LOD  ZERO                149   0183   69  0073  0126
         STO  ACCUM     DOONE     150   0126   24  0079  0132
DOONE    RAU  TERM   C            151   0132   60  7520  0175
         FAD  ACCUM               152   0175   32  0079  0105
         STU  ACCUM               153   0105   21  0079  0182
         LOD  INDXA               154   0182   69  0062  0215
         SXC   8001               155   0215   59  8001  0271
         NZC  REPET     DIDIT     156   0271   40  0274  0225
REPET    BMC  DIDIT     DOONE     157   0274   49  0225  0132
DIDIT    RAU  ACCUM               158   0225   60  0079  0233
         FSB  FLOX   A            159   0233   33  3220  0297
         STU  G      A            160   0297   21  3240  0093
         FMP  G      A            161   0093   39  3240  0140
         FDV  W      A            162   0140   34  3750  0400
         FAD  ERROR               163   0400   32  0085  0111
         STU  ERROR               164   0111   21  0085  0138
         SXC   0001               165   0138   59  0001  0044
         SXA   0001               166   0044   51  0001  0450
         NZA  READY     EXIT      167   0450   40  0221  0453
PRINT    STO  EXET                168   0500   24  0503  0406
         LOD            EOOCL     169   0406   69  0309  0012
         LOD  INDXB               170   0309   69  0412  0265
         RAB   8001     RESIT     171   0265   82  8001  0321
RESIT    LOD  EIGHT               172   0321   69  0024  0427
         RSA   8001     FILL      173   0427   81  8001  0283
FILL     LOD  BBEST  B            174   0283   69  5270  0123
         STO   1985  A            175   0123   24  3985  0188
         LOD  RBEST  B            176   0188   69  5290  0143
         STO   1986  A            177   0143   24  3986  0239
         SXB   0001               178   0239   53  0001  0345
         AXA   0002               179   0345   50  0002  0201
         NZB  NEXTB     WHOPE     180   0201   42  0104  0155
NEXTB    NZA  FILL      CAROZ     181   0104   40  0283  0108
CAROZ    PCH   1977               182   0108   71  1977  0477
         LOD  RESIT     EOOCL     183   0477   69  0321  0012
WHOPE    PCH   1977               184   0155   71  1977  0527
         LOD            EOOCL     185   0527   69  0280  0012
         LOD  ERMIN               186   0280   69  0333  0136
         STO   1984               187   0136   24  1984  0137
         PCH   1977     EXET      188   0137   71  1977  0503
PRIME    STO  TIXE                189   0550   24  0553  0456
         LOD  B      B            190   0456   69  5260  0013
         STO  BBEST  B            191   0013   24  5270  0173
         LOD  R      B            192   0173   69  5280  0383
         STO  RBEST  B            193   0383   24  5290  0193
         LOD  ERROR               194   0193   69  0085  0238
         STO  ERMIN     TIXE      195   0238   24  0333  0553
START    LOD  INDXB               196   1999   69  0412  0315
         RAB   8001     LOADY     197   0315   82  8001  0371
LOADY    LOD  BINIT  B            198   0371   69  5300  0603
         STO  B      B            199   0603   24  5260  0063
         STO  BBEST  B            200   0063   24  5270  0223
         LOD  RINIT  B            201   0223   69  5310  0113
         STO  R      B            202   0113   24  5280  0433
         STU  RBEST  B            203   0433   24  5290  0243
         SXB   0001               204   0243   53  0001  0149
         NZB  LOADY     SETUP     205   0149   42  0371  0653
SETUP    RAU  INDXA               206   0653   60  0062  0117
         RAA   8001               207   0117   80  8001  0273
         LOD  INDXB               208   0273   69  0412  0365
         RAB   8001               209   0365   82  8001  0421
         MPY   8001               210   0421   19  8001  0094
         STL  INDXC               211   0094   20  0224  0577
         RAC   8001     CALCA     212   0577   88  8001  0483
CALCA    LOD            TEMPP     213   0483   69  0186  0250
         LOD            TURMM     214   0186   69  0289  0300
         SXC   0001               215   0289   59  0001  0395
         SXA   0001               216   0395   51  0001  0251
         NZA  CALCA     CALCB     217   0251   40  0483  0205
CALCB    LOD  INDXA               218   0205   69  0062  0415
         RAA   8001               219   0415   80  8001  0471
         SXB   0001               220   0471   53  0001  0627
         NZB  CALCA     FILUP     221   0627   42  0483  0101
FILUP    LOD            ERR       222   0181   69  0134  0350
         LOD            PRIME     223   0134   69  0187  0550
         LOD            PRINT     224   0187   69  0190  0500
         LOD            PRNTM     225   0190   69  0293  0000
         LOD  INDXB               226   0293   69  0412  0465
         RAB   8001               227   0465   82  8001  0521
         LOD  INDXC               228   0521   69  0224  0677
         RAC   8001     LOOPB     229   0677   88  8001  0533
LOOPB    LOD  ZERO                230   0533   69  0073  0176
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         BTD  INDXF               231   0176   24 01     232
         RAU  BBEST  B            232   0232   60 527    275
         FAD  DELB   B  SUBB1     233   0275   32 580    727
BUBB1    BTU  B      B            234   0727   21 5260 0163
         LDD   8007               235   0163   69 8007 0119
         BTD  CTEMP               236   0119   24 0022 0325
         LDD   8006               237   0325   69 8006 0231
         STD  BTEMP               238   0231   24 0184 0237
         LDD  INDXA               239   0237   69 0062 0515
         RAA   8001     BUBB2     240   0515   80 8001 0571
BUBB2    LDD            TURMM     241   0571   69 0324 0300
         BXC   0001               242   0324   59 0001 0330
         SXA   0001               243   0330   51 0001 0236
         NZA  SUBB2     SUBB3     244   0236   40 0571 0240
SUBB3    LDD            ERR       245   0240   69 0343 0350
         RAU  ERMIN               246   0343   60 0333 0287
         FBB  ERRDR               247   0287   33 0085 0161
         RMI  BADDB     GOODR     248   0161   46 0064 0565
GODDR    LDD  BTEMP               249   0565   69 0184 0337
         RAB   8001               250   0337   82 8001 0393
         LDD  CTEMP               251   0393   69 0022 0375
         RAC   8001               252   0375   88 8001 0281
         RAU  BBEST  B            253   0281   60 5270 0425
         FSR  B      B            254   0425   33 5260 0387
         RMI  RPLUS     BMINS     255   0387   46 0290 0091
BPLUB    LDD            PRIME     256   0290   69 0443 0550
         LDD  FPONE               257   0443   69 0196 0199
         BTD  INDXF               258   0199   24 0129 0282
         STD  INDXD               259   0282   24 0135 0288
         RAU  BBEBT  B            260   0288   60 5270 0475
         FAD  DELB   B  SUBB1     261   0475   32 5800 0727
BMINB    LDD            PRIME     262   0091   69 0144 0550
         LDD  ZERO                263   0144   69 0073 0226
         BTD  INDXF               264   0226   24 0129 0332
         LDD  FPDNE               265   0332   69 0196 0249
         BTD  INDXO               266   0249   24 0135 0338
         RAU  BBEST  B            267   0338   60 5270 0525
         FSB  DELB   B  BUBB1     268   0525   33 5800 0727
BADDB    LDD  BTEMP               269   0064   69 0184 0437
         RAB   8001               270   0437   82 8001 0493
         RAU  INDXF               271   0493   60 0129 0583
         NZU  LDDPR     LETUP     272   0583   44 0487 0388
LETUP    LDD  BTEMP               273   0388   69 0184 0537
         RAB   8001               274   0537   82 8001 0543
         RAU  BBEST  B            275   0543   60 5270 0575
         FBB  B      B            276   0575   33 5260 0587
         BMI  MDREB     LODPR     277   0587   46 0340 0487
MDREB    LDD  BTEMP               278   0340   69 0184 0637
         RAB   8001               279   0637   82 8001 0593
         LDD  CTEMP               280   0593   69 0022 0625
         RAC   8001               281   0625   88 8001 0331
         RAU  BBEST  B            282   0331   60 5270 0675
         FSR  DELB   B  BUBB1     283   0675   33 5800 0727
LDDPR    LDD  ZERD                284   0487   69 0073 0276
         BTD  INDXF               285   0276   24 0129 0382
         LDD  FDUR                286   0382   69 0185 0438
         BTD  INDXZ               287   0438   24 0141 0194
         LDD  INDXD               288   0194   69 0135 0488
         BTD  INOXW               289   0488   24 0191 0244
         LDD  BBEST  R            290   0244   69 5270 0323
         STD  B      B            291   0323   24 5260 0213
         STD  BINIT  B            292   0213   24 5300 0703
         RAU  RBEST  B            293   0703   60 5290 0445
         FAD  DELR   B  BUBR1     294   0445   32 5810 0687
BUBR1    BTU  R      B            295   0687   21 5280 0633
         LDD  CTEMP               296   0633   69 0022 0725
         RAC   8001               297   0725   88 8001 0381
         LDD  BTEMP               298   0381   69 0184 0737
         RAB   8001               299   0737   82 8001 0643
         LDD  INDXA               300   0643   69 0062 0615
         RAA   8001     SUBR2     301   0615   80 8001 0621
BUBR2    LDD            TEMPP     302   0621   69 0374 0250
         LDD            TURMM     303   0374   69 0777 0300
         BXC   0001               304   0777   59 0001 0683
         BXA   0001               305   0683   51 0001 0339
         NZA  BUBR2     RDUT      306   0339   40 0621 0693
RDUT     RAU  RREBT  B            307   0693   60 5290 0495
         FSB  R      B            308   0495   33 5280 0107
         NZU  SUBR3     NEXT      309   0107   44 0211 0462
BUBR3    LDD            ERR       310   0211   69 0114 0350
         RAU  ERMIN               311   0114   60 0333 0787
         FSB  ERRDR               312   0787   33 0085 0261
         NZU  BTAY      PLAY      313   0261   44 0665 0016
PLAY     RAU  INDXZ               314   0016   60 0141 0545
         FBB  FPONE               315   0545   33 0196 0373
         BTU  INDXZ               316   0373   21 0141 0294
         NZU  STAY      PLYMR     317   0294   44 0665 0098
BTAY     BMI  BADDR     GODDR     318   0665   46 0118 0169
PLYMR    LDD  INDXW               319   0098   69 0191 0344
         STD  INDXO     NEXT      320   0344   24 0135 0462
GOODR    LDD  BTEMP               321   0169   69 0184 0837
         RAB   8001               322   0837   82 8001 0743
         LDD  CTEMP               323   0743   69 0022 0775
         RAC   8001               324   0775   88 8001 0431
         RAU  RBEBT  B            325   0431   60 5290 0595
         FBB  R      B            326   0595   33 5280 0157
         BMI  RPLUB     RMINS     327   0157   46 0060 0311
RPLUB    LDD            PRIME     328   0060   69 0263 0550
         LDD  FPONE               329   0263   69 0196 0299
         BTD  INDXF               330   0299   24 0129 0432
         BTD  INDXD               331   0432   24 0135 0538
         RAO  RBEST  B            332   0538   60 5290 0645
         FAD  DELR   B  SOBR1     333   0645   32 5810 0687
RMINB    LDD            PRIME     334   0311   69 0164 0550
         LDD  ZERO                335   0164   69 0073 0326
         STD  INDXF               336   0326   24 0129 0482
         LDD  FPDNE               337   0482   69 0196 0349
         BTD  INDXD               338   0349   24 0135 0588
         RAU  RBEBT  B            339   0588   60 5290 0695
         FBB  DELR   B  SOBR1     340   0695   33 5810 0687
BADDR    LDD  FDUR                341   0118   69 0185 0638
         BTD  INDXZ               342   0638   24 0141 0394
         RAU  INDXF               343   0394   60 0129 0733
         NZU  FIXUP     TEBTR     344   0733   44 0887 0688
TEBTR    LDD  BTEMP               345   0688   69 0184 0937
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         RAB  8001               346   0937   82 8001 0793
         RAU  RBEST B            347   0793   60 5290 0745
         FSB  R     B            348   0745   33 5280 0207
         BMI  MORER   FIXUP      349   0207   46 0110 0887
MORER    LDD  STEMP              350   0110   69 0184 0987
         RAB  8001               351   0987   82 8001 0843
         LDD  CTEMP              352   0843   69 0022 0825
         RAC  8001               353   0825   88 8001 0481
         RAU  RBEST B            354   0481   60 5290 0795
         FSB  DELR  B SUBR1      355   0795   33 5810 0687
FIXUP    RAU  RBEST B SUBR1      356   0887   60 5290 0687
NEXT     LDD  RBEST B            357   0462   69 5290 0893
         STD  RINIT B            358   0893   24 5310 0313
         SXB  0001               359   0313   53 0001 0219
         NZB  AGAIN   ENUFF      360   0219   42 0072 0423
AGAIN    RAU  CTEMP              361   0072   60 0022 0827
         SUP  INDXA              362   0827   11 0062 0167
         STU  CTEMP              363   0167   21 0022 0875
         RAU  BTEMP              364   0875   60 0184 0389
         SUP  ONE                365   0389   11 0392 0347
         STU  BTEMP              366   0347   21 0184 1037
         LDD  CTEMP              367   1037   69 0022 0925
         RAC  8001               368   0925   88 8001 0531
         LDD  BTEMP              369   0531   69 0184 1087
         RAB  8001    LOOPB      370   1087   82 8001 0533
ENUFF    LDD          PRINT      371   0423   69 0376 0500
         LDD  8000               372   0376   69 8000 0532
         RAU  8001               373   0532   60 8001 0439
         BMI  DEVEA   PLUSB      374   0439   46 0442 0943
PLUSB    RAU  INDXD              375   0943   60 0135 0489
         NZU  TIRED   TIGHT      376   0489   44 0993 0444
TIRED    LDD  INDXB              377   0993   69 0412 0715
         RAB  8001               378   0715   82 8001 0671
         LDD  INDXC              379   0671   69 0224 0877
         RAC  8001               380   0877   88 8001 0783
         LDD  ZERO               381   0783   69 0073 0426
         STD  INDXD   LOOPB      382   0426   24 0135 0533
TIGHT    LDD  INDXB              383   0444   69 0412 0765
         RAB  8001    RETTN      384   0765   82 8001 0721
RETTN    RAU  DELB  B            385   0721   60 5800 0255
         FDV  TEN                386   0255   34 0158 0208
         STU  DELB  B            387   0208   21 5800 0753
         RAU  DELR  B            388   0753   60 5810 0815
         FDV  TEN                389   0815   34 0158 0258
         STU  DELR  B            390   0258   21 5810 0363
         SXB  0001               391   0363   53 0001 0269
         NZB  RETTN   QUAAD      392   0269   42 0721 0473
QUAAD    RAB  0001               393   0473   82 0001 0179
         RAU  DELR  B            394   0179   60 5810 0865
         FSB  ONEHD              395   0865   33 0168 0845
         BMI  DEVEA   TIRED      396   0845   46 0442 0993
DEVEA    LDD          EDDCL      397   0442   69 0895 0012
         LDD  INDXA              398   0895   69 0062 0915
         RAA  8001    RESET      399   0915   80 8001 0771
RESET    LDD  EIGHT              400   0771   69 0024 0927
         RSB  8001    STUFF      401   0927   83 8001 0833
STUFF    LDD  Z     A            402   0833   69 3200 0803
         STD  1985  B            403   0803   24 5985 0738
         LDD  G     A            404   0738   69 3240 1043
         STD  1986  B            405   1043   24 5986 0539
         LDD  FLUX  A            406   0539   69 3220 0523
         STD  1987  B            407   0523   24 5987 0390
         SXA  0001               408   0390   51 0001 0246
         AXB  0004               409   0246   52 0004 0052
         NZA  MORES   FINIS      410   0052   40 0305 0506
MORES    NZB  STUFF   CARDS      411   0305   42 0833 0359
CARDS    PCH  1977               412   0359   71 1977 0977
         LDD  RESET   EDDCL      413   0977   69 0771 0012
FINIS    RAB  0001               414   0506   82 0001 0518
         RAU  DELR  B            415   0512   60 5810 0965
         FSB  ONEHD              416   0965   33 0168 0945
         BMI  END     SOMOR      417   0945   46 0148 0399
SOMOR    PCH  1977    PLUSB      418   0399   71 1977 0943
END      PCH  1977     8000      419   0148   71 1977 8000
                                       0073   00 0000 0000
                                       0196   10 0000 0051
                                       0185   40 0000 0051
                                       0158   10 0000 0052
                                       0090   10 0000 0053
                                       0392   00 0000 0001
                                       0024   00 0000 0008
```

APPENDIX C

Description and Explanation of the IBM-650 Computer Program Used for Fitting Empirically Experimental Data with an Even Fourier Series

The computer code was written to fit an even Fourier series to the experimentally determined surface temperature traces during a transient burst. The program was written in SOAP II and floating point form. The object program is listed and the logic diagram is shown in this appendix.

The data were fit empirically by a finite number of terms of the even trigonometric series.

$$\theta(t) = \sum_{i=1}^{p} B_i \cos \frac{2\pi i t}{a} , \qquad \text{(C-1)}$$

where

$$B_o = 2/a \int_o^a y(t)\, dt$$

and

$$B_i = 1/a \int_o^a y(t) \cos \frac{2\pi i t}{a}\, dt .$$

The integrations were carried out numerically by means of Simpson's rule thus requiring an odd number of data points.

The program input consisted of the experimental data and their respective times, the period, the time increment between data points and a specification of the number of terms. These data were read into the machine on one-word load cards. Each one-word load card contained a particular constant or piece of data and its specific storage location. Table C-1 lists the input data needed for this program. Storage locations limit the product of the number of terms and the number of data points to

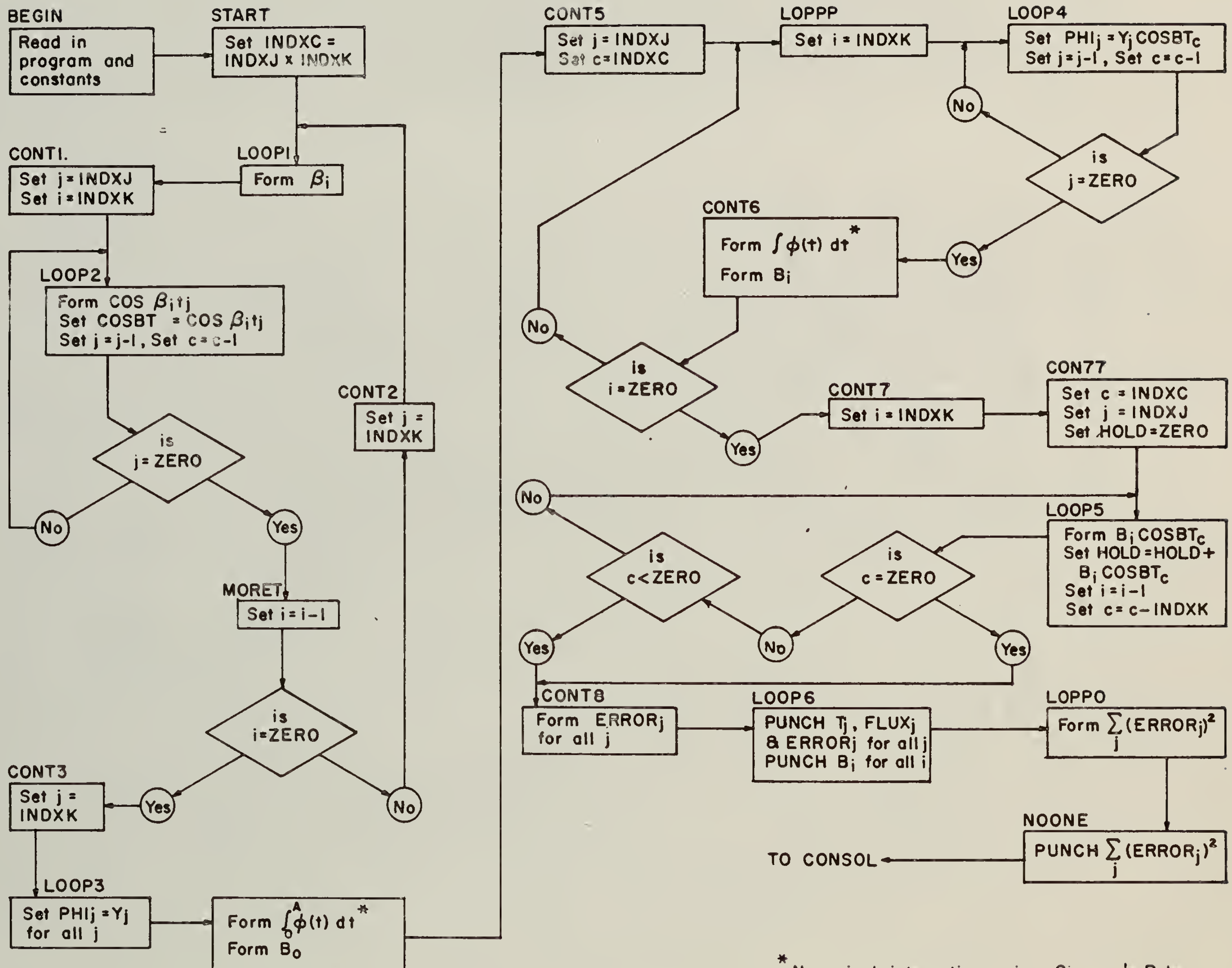
less than 550. The number of terms and the number of data points are each limited to less than 50.

Table C-1. Input data required for use of the IBM-650 program which fits empirically experimental data with a finite number of terms of an even Fourier series.

Symbol	Expanation	Storage Location
ZERO	0.00	0083
FPONE	1.00	0034
FPTWO	2.00	0108
FPTRE	3.00	0207
FPFOR	4.00	0008
PI	3.14159	0261
INDX2	Index Number 2 (0000000002)	0160
INDXJ	Number of Terms (00000000xx)	1001
INDXK	Number of Data Points (00000000xx)	1002
A	Period of Cosine Terms	1003
H	Time Increment Between Data Points	1004
T_i	Time of ith data point	(1100 + i)
Y_i	Experimental data at ith point	(1150 + i)

The machine punched out a card having an eight word capacity, each work consisting of 10 digits and a sign. The first output consisted of the time, the calculated value and the residual between the calculated values and the experimental data for the last data point in word locations 1, 2 and 3, respectively. The same information for the next to last data point was punched out in columns 5, 6 and 7 of the same card. The same information for the two previous data points was punched out on the next card. The above procedure was continued until the time, residuals and calculated values were punched for each data point. $\underline{B_1}$ was then stored in word location 1, $\underline{B_2}$ in word location two, etc. until all of the <u>B's</u>

had been stored and punched. If more than 8 B's were calculated the first 8 were stored and a card punched. Additional B's were punched in succeeding cards with the lower number B's starting on the left of each card. Finally the summation of the square of the residual at each data point was punched out in word location 8 of a final card.



* Numerical integration using Simpson's Rule.

LOGIC DIAGRAM-APPENDIX C

OBJECT PROGRAM-APPENDIX C

```
               BLR   1000      1999                   1    0000    00  0000  0000
               SYN  ERROR      1050                   2    0000    00  0000  0000
               BLR   0977      0985                   3    0000    00  0000  0000
               SYN  T          1100     T1 SMALL      4    0000    00  0000  0000
               SYN  Y          1150     Y1 WITH T1    5    0000    00  0000  0000
               SYN  S          1200                   6    0000    00  0000  0000
               SYN  STA        1250                   7    0000    00  0000  0000
               SYN  PHI        1300                   8    0000    00  0000  0000
               SYN  FLUX       1350                   9    0000    00  0000  0000
               SYN  COSST      1400                  10    0000    00  0000  0000
               SYN  START      1000                  11    0000    00  0000  0000
               SYN  INDXJ      1001     NO TERMS     12    0000    00  0000  0000
               SYN 'INOXK      1002     NO DATA PT   13    0000    00  0000  0000
               SYN  A          1003                  14    0000    00  0000  0000
               SYN  H          1004                  15    0000    00  0000  0000
     EOOCR     STO  EXIT                             16    0000    24  0003  0006
               BMI  NEGAT      REDUC                 17    0006    46  0009  0010
     NEGAT     FAD  TWOPI                            18    0009    32  0012  0039
               BMI  NEGAT                            19    0039    46  0009  0043
               FSB  ONEPI      COSIO                 20    0043    33  0046  0023
     REDUC     FSB  TWOPI                            21    0010    33  0012  0089
               BMI             REOUC                 22    0089    46  0042  0010
               FAD  ONEPI      COSIU                 23    0042    32  0046  0023
     COSIO     STU  THETA                            24    0023    21  0028  0031
               RSU  FPONE                            25    0031    61  0034  0139
               STU  TERMM                            26    0139    21  0044  0047
               STU  FUNKT                            27    0047    21  0002  0005
               STL  ENN        NEGST                 28    0005    20  0059  0062
     EOOSR     STO  EXIT                             29    0050    24  0003  0056
               BMI  NEGAV      REOUD                 30    0056    46  0109  0060
     NEGAV     FAD  TWOPI                            31    0109    32  0012  0189
               BMI  NEGAV                            32   '0189    46  0109  0093
               FSB  ONEPI      SINET                 33    0093    33  0046  0073
     REDUU     FSB  TWOPI                            34    0060    33  0012  0239
               BMI             REDUD                 35    0239    46  0092  0060
               FAD  ONEPI      SINET                 36    0092    32  0046  0073
     SINET     STU  THETA                            37    0073    21  0028  0001
               RSU   8003                            38    0081    61  8003  0289
               STU  TERMM                            39    0289    21  0044  0097
               STU  FUNKT                            40    0097    21  0002  0055
               LOD  FPONE                            41    0055    69  0034  0037
               STO  ENN        NEGST                 42    0037    24  0059  0062
     NEGST     RAU  ENN                              43    0062    60  0059  0013
               FAD  FPONE                            44    0013    32  0034  0011
               STU  NPONE                            45    0011    21  0016  0019
               FAD  FPONE                            46    0019    32  0034  0061
               STU  ENN                              47    0061    21  0059  0112
               RSU  TERMM                            48    0112    61  0044  0049
               FMP  THETA                            49    0049    39  0028  0078
               FMP  THETA                            50    0078    39  0028  0128
               FDV  NPONE                            51    0128    34  0016  0066
               FDV  ENN                              52    0066    34  0059  0159
               STU  TERMM                            53    0159    21  0044  0147
               RAM  FUNKT                            54    0147    67  0002  0007
               STL  FMAG                             55    0007    20  0111  0014
               RAM  TERMM                            56    0014    67  0044  0099
               RAU   8002                            57    0099    60  8002  0057
               FDV  FMAG                             58    0057    34  0111  0161
               FSB  SIZES                            59    0161    33  0064  0041
               BMI  ENUFF                            60    0041    46  0094  0045
               RAU  FUNKT                            61    0045    60  0002  0107
               FAD  TERMM                            62    0107    32  0044  0021
               STU  FUNKT      NEGST                 63    0021    21  0002  0062
     ENUFF     RAU  FUNKT      EXIT                  64    0094    60  0002  0003
     SIZES      10   0000       0043                 65    0064    10  0000  0043
     TWOPI      62   8318       5351                 66    0012    62  8318  5351
     ONEPI      31   4159       2751                 67    0046    31  4159  2751
     FPONE      10   0000       0051                 68    0034    10  0000  0051
     COS       STO  EXITC                            69    0100    24 '0053  0106
               RAU  STA     A                        70    0106    60  3250  0105
               FMP  T       S                        71    0105    39  5100  0150
               LDD             EOOCH    BUSLDSUP     72    0150    69  0103  0000
               STU  COSST      EXITC                 73    0103    21  7400  0053
     INTGT     STD  EGSIT                            74    0200    24  0153  0156
               RAB   0001                            75    0156    82  0001  0162
               RAU  PHI     S                        76    0162    60  5300  0155
               STU  ACCUM      LOP33                 77    0155    21  0110  0063
     LOP33     AXB   0001                            78    0063    52  0001  0069
               RAU  PHI     S                        79    0069    60  5300  0205
               FMP  FPFOR               INPUT        80    0205    39  0008  0058
               FAD  ACCUM                            81    0058    32  0110  0087
               STU  ACCUM                            82    0087    21  0110  0113
               AXB   0001                            83    0113    52  0001  0119
               RAU  PHI     S                        84    0119    60  5300  0255
               FMP  FPTWO               INPUT        85    0255    39  0108  0158
               FAD  ACCUM                            86    0158    32  0110  0137
               STU  ACCUM                            87    0137    21  0110  0163
               RAU   8006                            88    0163    60  8006  0071
               SUP  INDXK                            89    0071    11  1002  0157
               AUP  INOX2                  INPUT     90    0157    10  0160  0015
               NZU  LOP33      GO111                 91    0015    44  0063  0020
     GO111     AXB   0001                            92    0020    52  0001  0026
               RAU  PHI     S                        93    0026    60  5300  0305
               FMP  FPFOR                            94    0305    39  0008  0208
               FAD  ACCUM                            95    0208    32  0110  0187
               STU  ACCUM                            96    0187    21  0110  0213
               AXB   0001                            97    0213    52  0001  0169
               RAU  PHI     S                        98    0169    60  5300  0355
               FAD  ACCUM                            99    0355    32  0110  0237
               FMP  H                   INPUT       100    0237    39  1004  0004
               FDV  FPTRE      EGSIT    INPUT       101    0004    34  0207  0153
     EOOCL     STO  ZZZ1                EXIT INSTR  102    0250    24  0203  0206
               LDD  ZZZ10                           103    0206    69  0209  0212
               STD   0977                           104    0212    24  0977  0030
               STD   0978                           105    0030    24  0978  0131
               STD   0979                           106    0131    24  0979  0032
               STD   0980                           107    0032    24  0980  0033
               STD   0981                           108    0033    24  0981  0084
               STD   0982                           109    0084    24  0982  0035
               STD   0983                           110    0035    24  0983  0036
               STD   0984      ZZZ1                 111    0036    24  0984  0203
     ZZZ10      00   0000       0000                112    0209    00  0000  0000
     START     RAU  INDXK                           113    1000    60  1002  0257
               MPY  INDXJ                           114    0257    19  1001  0022
               STL  INDXC                           115    0022    20  0027  0080
```

```
         LDD ZERO             INPUT       116  0080  69 0083 0086
         RAA  8001                        117  0086  80 8001 0142
         STD N         LOOP1              118  0142  24 0095 0048
LOOP1    AXA  0001                        119  0048  50 0001 0054
         RAU N                            120  0054  60 0095 0149
         FAD FPONE            INPUT       121  0149  32 0034 0211
         STU N                            122  0211  21 0095 0098
         FMP FPTWO            INPUT       123  0098  39 0108 0258
         FMP PI               INPUT       124  0258  39 0261 0311
         FDV A                INPUT       125  0311  34 1003 0253
         STU BTA     A                    126  0253  21 3250 0303
         RAU  8005                        127  0303  60 8005 0361
         SUP INDXJ                        128  0361  11 1001 0405
         NZU LOOP1     CONT1              129  0405  44 0048 0210
CONT1    LDD INDXJ                        130  0210  69 1001 0104
         RAA  8001                        131  0104  80 8001 0260
         LDD INDXK                        132  0260  69 1002 0455
         RAB  8001                        133  0455  82 8001 0411
         LDD INDXC                        134  0411  69 0027 0130
         RAC  8001     LOOP2              135  0130  88 8001 0136
LOOP2    LDD           COS    SUBROUTINE  136  0136  69 0339 0100
         SXA  0001                        137  0339  53 0001 0145
         SXC  0001                        138  0145  59 0001 0001
         NZB LOOP2     MORET              139  0001  42 0136 0505
MORET    SXA  0001                        140  0505  51 0001 0461
         NZA CONT2     CONT3              141  0461  40 0114 0065
CONT2    LDD INDXK                        142  0114  69 1002 0555
         AXB  8001     LOOP2              143  0555  52 8001 0136
CONT3    RAA  0000                        144  0065  80 0000 0121
         LDD INDXK                        145  0121  69 1002 0605
         AXB  8001     LOOP3              146  0605  52 8001 0511
LOOP3   'LDD Y       B                    147  0511  69 5150 0353
         STD PHI     B                    148  0353  24 5300 0403
         SXB  0001                        149  0403  53 0001 0259
         NZB LOOP3     CONT4              150  0259  42 0511 0263
CONT4    LDD           INTGT  SUBROUTINE  151  0263  69 0116 0200
         FMP FPTWO                        152  0116  39 0108 0308
         FDV A                            153  0308  34 1003 0453
         STU BZERO     CONT5              154  0453  21 0358 0561
CONT5    RAA  0000                        155  0561  80 0000 0017
         RAB  0000                        156  0017  82 0000 0123
         RAC  0000     CON55              157  0123  88 0000 0029
CON55    LDD INDXC                        158  0029  69 0027 0180
         RAC  8001                        159  0180  88 8001 0186
         LDD INDXJ                        160  0186  69 1001 0154
         RAA  8001     LOPPP              161  0154  80 8001 0310
LOPPP    LDD INDXK                        162  0310  69 1002 0655
         RAB  8001     LOOP4              163  0655  82 8001 0611
LOOP4    RAU Y       B                    164  0611  60 5150 0705
         FMP COSBT   C                    165  0705  39 7400 0300
         STU PHI     B                    166  0300  21 5300 0503
         SXB  0001                        167  0503  53 0001 0309
         SXC  0001                        168  0309  59 0001 0115
         NZB LOOP4     CONT6              169  0115  42 0611 0219
CONT6    LDD           INTDT              170  0219  69 0072 0200
         FMP FPFOR                        171  0072  39 0008 0408
         FDV A                            172  0408  34 1003 0553
         STU B       A                    173  0553  21 3200 0603
         SXA  0001                        174  0603  51 0001 0359
         NZA LOPPP     CONT7              175  0359  40 0310 0313
CONT7    RAA  0000                        176  0313  80 0000 0269
         RAB  0000                        177  0269  82 0000 0025
         LDD INDXK                        178  0025  69 1002 0755
         AXB  8001                        179  0755  52 8001 0661
         LDD INDXJ                        180  0661  69 1001 0204
         AXA  8001                        181  0204  50 8001 0360
         RAC  0000     CON77              182  0360  88 0000 0166
CON77    LDD INDXC                        183  0166  69 0027 0230
         AXC  8001                        184  0230  58 8001 0236
         LDD INDXJ                        185  0236  69 1001 0254
         RAA  8001                        186  0254  80 0001 0410
         LDD ZERO                         187  0410  69 0083 0286
         STD HOLD      LOOP5              188  0286  24 0389 0192
LOOP5    RAU B       A                    189  0192  60 3200 0805
         FMP COSBT   C                    190  0805  39 7400 0350
         FAD HOLD                         191  0350  32 0389 0165
         STU HOLD                         192  0165  21 0389 0242
         SXA  0001                        193  0242  51 0001 0148
         LDD INDXK                        194  0148  69 1002 0855
         SXC  8001                        195  0855  59 8001 0711
         NZC           CONT8              196  0711  48 0164 0215
         BMC CONT8     LOOP5              197  0164  49 0215 0192
CONT8    RAU HOLD                         198  0215  60 0389 0143
         FAD BZERO                        199  0143  32 0358 0085
         STU FLUX    B                    200  0085  21 5350 0653
         FSB Y       B                    201  0653  33 5150 0077
         STU ERROR   B                    202  0077  21 5050 0703
         SXB  0001                        203  0703  53 0001 0409
         SXC  0001                        204  0409  59 0001 0265
         NZB CON77     PRINT              205  0265  42 0166 0319
PRINT    LDD INDXK                        206  0319  69 1002 0905
         AXB  8001     LOOP7              207  0905  52 8001 0761
LOOP7    LDD           EOOCL  SUBROUTINE  208  0761  69 0214 0250
         RSA  0008     LOOP6              209  0214  81 0008 0070
LOOP6    LDD T       B                    210  0070  69 5100 0753
         STD  0985   A                    211  0753  24 2985 0038
         LDD FLUX    B                    212  0038  69 5350 0803
         STD  0986   A                    213  0803  24 2986 0439
         LDD ERROR   B                    214  0439  69 5050 0853
         STD  0987   A                    215  0853  24 2987 0040
         AXA  0004                        216  0040  50 0004 0096
         SXB  0001                        217  0096  53 0001 0052
         NZB           FINIS              218  0052  42 0955 0256
         NZA LOOP6     CONT9              219  0955  40 0070 0459
CONT9    PCH  0977     LOOP7              220  0459  71 0977 0761
FINIS    PCH  0977                        221  0256  71 0977 0127
         RSB  0007                        222  0127  83 0007 0133
         LDD           EOOCL              223  0133  69 0336 0250
         LDD BZERO                        224  0336  69 0358 0811
         STD  0977                        225  0811  24 0977 0280
         RAA  0001     LOOP8              226  0280  80 0001 0386
LOOP8    LDD B       A                    227  0386  69 3200 0903
         STD  0985   B                    228  0903  24 4985 0088
         AXA  0001                        229  0088  50 0001 0144
         AXB  0001                        230  0144  52 0001 0400
```

```
         RAU  8006                  231    0400    60 8005 0307
         SUB  INDXJ                 232    0307    11 1001 0306
         NZU             FINAL      233    0306    44 0509 0460
         NZB  LOOPB      MOST       234    0509    42 0386 0363
MOST     PCH   0977                 235    0363    71 0977 0177
         LDD             EOOCL      236    0177    69 0330 0250
         RSB   0007      LOOPB      237    0330    83 0007 0386
FINAL    LDD  B      A              238    0460    69 3200 0953
         STD   0985  8              239    0953    24 4985 0138
         PCH   0977                 240    0138    71 0977 0227
         LDD  INDXK                 241    0227    69 1002 0356
         XAB   8001                 242    0356    82 8001 0262
         LDD  ZERO                  243    0262    69 0083 0436
         STD  SOMME      LOPPU      244    0436    24 0489 0292
LOPPO    RAU  ERROR  8              245    0292    60 5050 0406
         FMP  ERROR  8              246    0406    39 5050 0450
         FAD  SOMME                 247    0450    32 0489 0315
         STU  SOMME                 248    0315    21 0489 0342
         SXB   0001                 249    0342    53 0001 0198
         NZB  LOPPO      NDONE      250    0198    42 0292 0102
NOONE    LDD             EOOCL      251    0102    69 0456 0250
         LDD  SOMME                 252    0456    69 0489 0392
         STD   0984                 253    0392    24 0984 0287
         PCH   0977       8000      254    0287    71 0977 8000
                                           0083    00 0000 0000
                                           0034    10 0000 0051
                                           0108    20 0000 0051
                                           0207    30 0000 0051
                                           0008    40 0000 0051
                                           0160    00 0000 0002
                                           0261    31 4159 2751
```

APPENDIX D

Description and Explanation of the IBM-650 Computer Program Used to Calculate Temperature Distributions.

The computer program was written to calculate the temperature distribution in a unit cell of a nuclear reactor system given the heat generation rate and fuel element surface temperature as a function of time. The temperature rise over the initial temperature is given by

$$\theta_f(x,t) = \sum_{i=1}^{p} B_i Z_i^{\frac{1}{2}} \cos(\beta_i t + \varphi_i) - \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\left(\frac{n\pi x}{2L}\right) e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{(L^2/n\pi\alpha)\left(\sin\frac{n\pi}{2}\right)}$$

$$\times \left\{ \sum_{i=1}^{p} \frac{B_i \left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^2} + \beta_i^2} + \sum_{j=1}^{s} \frac{q_{oo}\alpha A_j \cosh \kappa L}{k\left(\frac{n^2\pi^2\alpha}{4L^2}+\lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)} \right\} \qquad \text{(D-1)}$$

$$+ \sum_{j=1}^{s} \frac{q_{oo}\alpha A_j e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)} \left\{ \frac{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, x\right)\cosh \kappa x}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)} - \cosh \kappa x \right\} .$$

in the fuel and by

$$\theta_m(x,t) = \sum_{i=1}^{p} B_i Z_i^{\frac{1}{2}} \cos(\beta_i t + \varphi_i) + \sum_{n=1,3,5,\cdots}^{\infty} \frac{\cos\left(\frac{n\pi x}{2L}\right) e^{-\frac{n^2\pi^2\alpha}{4L^2}t}}{(L^2/n\pi\alpha)\left(\sin\frac{n\pi}{2}\right)}$$

$$\times\left\{\sum_{i=1}^{p}\frac{B_i\left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^4}+\beta_i} - \sum_{j=1}^{s}\frac{F\,\alpha\,A_j}{k\left(\frac{n^2\pi^2\alpha}{4L^2}+\lambda_j\right)}\right\} \tag{D-2}$$

$$+\sum_{j=1}^{s}\frac{F\,\alpha\,A_j\,e^{\rho_j t}}{k\,(-\lambda_j)}\left\{\frac{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,x}{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,L}-1\right\}$$

in the moderator. The moderator equation is obtained from the fuel temperature distribution by setting $\underline{q_{oo}}$ equal to $\underline{F}$ and $\underline{\kappa}$ equal to zero.

The equivalence between elements of the algebraic equations and the symbolic logic of the computer program is shown in Table D-1.

Table D-1. Definition of symbolic terms of the IBM-650 computer program for calculating temperature distributions.

$A1_i = B_i\, Z_i(x) \cos \beta_i t$

$A3P_j = q_{oo}\,\alpha\, A_j\, e^{\lambda_j t} / k\,(\alpha\kappa^2 - \lambda_j)$

$$A3SUM_j = \frac{\cosh \kappa L \quad \cosh\sqrt{\frac{\lambda_j}{\alpha}}\,x}{\cosh\sqrt{\frac{\lambda_j}{\alpha}}\,L} - \cosh \kappa x$$

$A3_j = (A3P_j)\,(A3SUM_j)$

$CSHLL_j = \cosh\sqrt{\frac{\lambda_j}{\alpha}}\,L$

$CSHLX_j = \cosh\sqrt{\frac{\lambda_j}{\alpha}}\,x$

$COSLL_j = \cos\sqrt{\frac{\lambda_j}{\alpha}}\,L$

$COSLX_j = \cos\sqrt{\frac{\lambda_j}{\alpha}}\,x$

Table D-1 cont.

$$ARG1_n = n\pi \quad 2L$$

$$ARG2_n = n^2\pi^2\alpha \quad 4L^2$$

$$A2ST1_i = B_i \left(\frac{n^2\pi^2\alpha}{4L^2}\right) \bigg/ \left(\frac{n^4\pi^4\alpha^2}{16L^2} + \beta_i^2\right)$$

$$A2ST1_j = q_\infty \alpha A_j' \cosh \kappa L / k \left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right) \left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)$$

$$A2DDT_n = \frac{\cos \frac{n\pi x}{2L} \quad e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{\left(\frac{L^2}{n\pi\alpha}\right) \sin \frac{n\pi}{2}}$$

$$A2SUM_n = \left(\sum_{i=1}^{p} A2ST1_i + \sum_{j=1}^{s} A2ST1_j\right)_n$$

$$TERM_n = (A2DDT_n)(A2SUM_n)$$

Table D-2. Input Data Required for Use of the IBM-650 Computer Program Used to Calculate Temperature Distributions.

Symbol	Explanation	Storage Location
ZERO	0.00	0264
ONE	1.00	0662
TWO	2.00	0520
PI	3.14159	0018
FIFTY	50.00	0361
CRIT	0.0001	0788
ALPHA	Thermal Diffusivity	0278

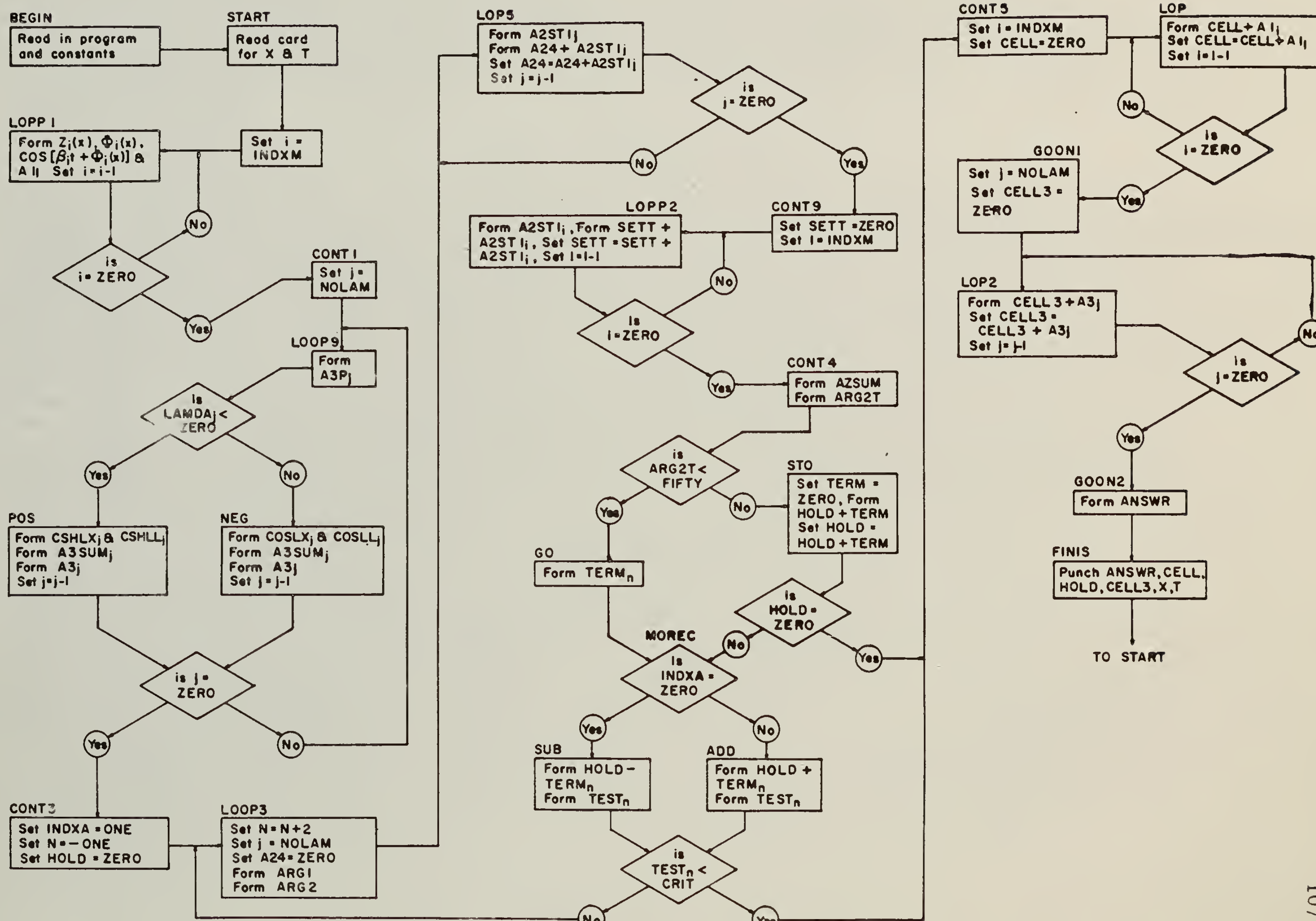
Table D-2 cont.

Symbol	Explanation	Storage Location
KAPPA	Reciprocal of Thermal Neutron Diffusion Length in Fuel	0436
Qoo	Normalization Factor for Heat Generation	0324
KAY	Thermal Conductivity	0581
L	Half-Thickness of Region	0186
INDXM	No. of Terms, Surface Temperature Fit (00000000xx)	0076
NOLAM	No. of Terms, Heat Generation Fit (00000000xx)	0456
$AYEJ_j$	Amplitude Parameter, Heat Generation Fit, A_j	(0200 + j)
$LAMDA_j$	Exponential Parameter, Heat Generation Fit, $_j$	(0220 + j)
$AMMM_i$	Amplitude Parameter, Surface Temperature Fit, B_i	(0100 + i)
$BTAA_i$	Period Parameter Surface Temperature Fit, B_i	(0120 + i)

The output from this program is punched out on one card having an eight word capacity, one word consisting of 10 digits and a sign. The form of the output is shown in Table D-3.

Table D-3. Output Form for IBM-650 Computer Program Used to Calculate Temperature Distribution

WORD 1	WORD 2	WORD 3	WORD 4	WORD 5	WORD 6	WORD 7	WORD 8
$\theta(x,t)$	$\sum_{i=1}^{p} A1_i$	$\sum_{n=1,3,5}^{\infty} (Term)_n$	$\sum_{j=1}^{s} A3_j$	x	t	--	--



LOGIC DIAGRAM-APPENDIX D

OBJECT PROGRAM-APPENDIX D

	BLR	0200	0860	1	0000	00	0000	0000	-
	BLR	1800	1899	2	0000	00	0000	0000	-
	SYN	ARTAN	1800	3	0000	00	0000	0000	-
	SYN	START	1999	4	0000	00	0000	0000	-
	SYN	AYEJ	0200	5	0000	00	0000	0000	-
	SYN	LAMDA	0280	6	0000	00	0000	0000	-
	SYN	A3	0240	7	0000	00	0000	0000	-
	BLR	1951	1960	8	0000	00	0000	0000	-
	BLR	1977	1984	9	0000	00	0000	0000	-
	BLR	0100	0160	10	0000	00	0000	0000	-
	SYN	AMMM	0100	11	0000	00	0000	0000	-
	SYN	BTAA	0120	12	0000	00	0000	0000	-
	SYN	ALPHA	0278	13	0000	00	0000	0000	-
	SYN	KAPPA	0436	14	0000	00	0000	0000	-
	SYN	L	0186	15	0000	00	0000	0000	-
	SYN	Q00	0324	16	0000	00	0000	0000	-
	SYN	KAY	0581	17	0000	00	0000	0000	-
	SYN	INOXM	0076	18	0000	00	0000	0000	-
	SYN	NOLAM	0456	19	0000	00	0000	0000	-
	SYN	A1	0140	20	0000	00	0000	0000	-
PHIS8	STD	ABCD2		21	0000	24	0003	0006	
	RAU	FCL		22	0006	60	0009	0013	
	FMP	FCX		23	0013	39	0016	0066	
	STU	DENN1		24	0066	21	0020	0023	
	RAU	FSL		25	0023	60	0026	0031	
	FMP	FSX		26	0031	39	0034	0084	
	FAD	DENN1		27	0084	32	0020	0047	
	STU	DN		28	0047	21	0002	0005	
	RAU	FSL		29	0005	60	0026	0081	
	FMP	FCX		30	0081	39	0016	0166	
	STU	NUMM2		31	0166	21	0070	0073	
	RAU	FCL		32	0073	60	0009	0063	
	FMP	FSX		33	0063	39	0034	0184	
	FSB	NUMM2		34	0184	33	0070	0097	
	FDV	DN		35	0097	34	0002	0052	
	LDD		ARTAN	36	0052	69	0055	1800	
	STU	THETO		37	0055	21	0010	0163	
	RAU	DN		38	0163	60	0002	0007	
	BMI		ABCD2	39	0007	46	0060	0003	
	RAU	THETO		40	0060	60	0010	0015	
	FAD	PI		41	0015	32	0018	0045	
	STU	THETO	ABCD2	42	0045	21	0010	0003	
EODEA	STD	AAA1		43	0050	24	0053	0056	
	STU	AAA14		44	0056	21	0310	0263	
	RAU	AAA16		45	0263	60	0266	0021	
	FAM	AAA14		46	0021	37	0310	0037	
	STU	AAA2		47	0037	21	0042	0095	
	LDD	AAA3		48	0095	69	0040	0001	
	STU	AAA4	AAA8	49	0001	24	0004	0057	
AAA8	RAU	AAA2		50	0057	60	0042	0197	
	FSB	AAA5		51	0197	33	0300	0027	
	BMI	AAA6		52	0027	46	0030	0181	
	STU	AAA2		53	0181	21	0042	0195	
	RAU	AAA4		54	0195	60	0004	0059	
	FMP	AAA7		55	0059	39	0012	0062	
	STU	AAA4	AAA8	56	0062	21	0004	0057	
AAA6	RAU	AAA2		57	0030	60	0042	0297	
	FSB	AAA3		58	0297	33	0048	0025	
	BMI	AAA28		59	0025	46	0028	0029	
	STU	AAA2		60	0029	21	0043	0295	
	RAU	AAA4		61	0295	60	0004	0309	
	FMP	AAA9		62	0309	39	0162	0262	
	STU	AAA4	AAA6	63	0262	21	0004	0030	
AAA28	RAU	AAA2		64	0028	60	0042	0347	
	FSB	AAA10		65	0347	33	0350	0077	
	BMI	AAA11		66	0077	46	0080	0281	
	STU	AAA2		67	0281	21	0042	0345	
	RAU	AAA4		68	0345	60	0004	0359	
	FMP	AAA12		69	0359	39	0312	0362	
	STU	AAA4	AAA28	70	0362	21	0004	0028	
AAA11	RAU	AAA2		71	0080	60	0042	0397	
	LDD		AAA17	72	0397	69	0400	0303	
	FMP	AAA4		73	0400	39	0004	0054	
	STU	AAA13		74	0054	21	0008	0011	
	RAU	AAA14		75	0011	60	0310	0065	
	BMI	AAA15		76	0065	46	0068	0019	
	RAU	AAA13	AAA1	77	0019	60	0008	0053	
AAA15	RAU	AAA3		78	0068	60	0048	0353	
	FDV	AAA13	AAA1	79	0353	34	0008	0053	
AAA17	STD	AAA18		80	0303	24	0306	0409	
	RAU	AAA3		81	0409	60	0048	0403	
	FAD	AAA2		82	0403	32	0042	0069	
	STU	AAA19		83	0069	21	0024	0177	
	LDD	AAA27		84	0177	69	0180	0033	
	STD	AAA20		85	0033	24	0036	0039	
	STD	AAA21		86	0039	24	0092	0395	
	RAU	AAA2		87	0395	60	0042	0447	
	FMP	AAA2		88	0447	39	0042	0192	
	STU	AAA23	AAA22	89	0192	21	0046	0049	
AAA22	FDV	AAA21		90	0049	34	0092	0292	
	STU	AAA24		91	0292	21	0096	0099	
	FAD	AAA19		92	0099	32	0024	0051	
	STU	AAA19		93	0051	21	0024	0277	
	RAU	AAA24		94	0277	60	0096	0301	
	FDV	AAA19		95	0301	34	0024	0074	
	FSB	AAA25		96	0074	33	0327	0453	
	BMI		AAA26	97	0453	46	0356	0307	
	RAU	AAA19	AAA18	98	0356	60	0024	0306	
AAA26	RAU	AAA20		99	0307	60	0036	0041	
	FAD	AAA3		100	0041	32	0048	0075	
	STU	AAA20		101	0075	21	0036	0089	
	FMP	AAA21		102	0089	39	0092	0342	
	STU	AAA21		103	0342	21	0092	0445	
	RAU	AAA23		104	0445	60	0046	0351	
	FMP	AAA2		105	0351	39	0042	0392	
	STU	AAA23	AAA22	106	0392	21	0046	0049	
AAA3	10	0000	0051	107	0048	10	0000	0051	
AAA5	50	0000	0051	108	0300	50	0000	0051	
AAA7	14	8410	0053	109	0012	14	8410	0053	
AAA9	27	1830	0051	110	0162	27	1830	0051	
AAA10	20	0000	0050	111	0350	20	0000	0050	
AAA12	12	2140	0051	112	0312	12	2140	0051	
AAA16	00	0000	0000	113	0266	00	0000	0000	
AAA25	70	0000	0047	114	0327	70	0000	0047	
AAA27	20	0000	0051	115	0180	20	0000	0051	

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ZMXSH   STD  ABCD1              0450   24  0503  0406
        RAU  ARGGX              0406   60  0459  0313
        LDD         COSHX       0313   69  0316  0169
        STU  CHAAX              0316   21  0170  0173
        RAU  ARGGX              0173   60  0459  0363
        LDD         EOOCR       0363   69  0366  0269
        STU  CSAAX              0366   21  0270  0273
        RAU  ARGGX              0273   60  0459  0413
        LDD         SINHX       0413   69  0416  0319
        STU  SHAAX              0416   21  0320  0323
        RAU  ARGGX              0323   60  0459  0463
        LDD         EOOSH       0463   69  0466  0369
        STU  SNAAX              0466   21  0370  0373
        RAU  ARGGL              0373   60  0176  0331
        LDD         COSHX       0331   69  0284  0169
        STU  CHAAL              0284   21  0038  0091
        RAU  ARGGL              0091   60  0176  0381
        LDD         EOOCH       0381   69  0334  0269
        STU  CSAAL              0334   21  0088  0191
        RAU  ARGGL              0191   60  0176  0431
        LDD         SINHX       0431   69  0384  0319
        STU  SHAAL              0384   21  0188  0291
        RAU  ARGGL              0291   60  0176  0481
        LDD         EOOSH       0481   69  0434  0369
        STU  SNAAL              0434   21  0288  0341
        RAU  CHAAL              0341   60  0038  0043
        FMP  CSAAL              0043   39  0088  0338
        STU  FCL                0338   21  0009  0412
        FMP  CHAAL              0412   39  0038  0388
        FMP  CSAAL              0388   39  0088  0438
        STU  DEIIN              0438   21  0442  0495
        RAU  SNAAL              0495   60  0288  0093
        FMP  SHAAL              0093   39  0188  0488
        STU  FSL                0488   21  0026  0079
        FMP  SNAAL              0079   39  0288  0538
        FMP  SHAAL              0538   39  0188  0588
        FAD  DEIIN              0588   32  0442  0419
        STU  DENNM              0419   21  0174  0377
        RAU  CHAAX              0377   60  0170  0175
        FMP  CSAAX              0175   39  0270  0420
        STU  FCX                0420   21  0016  0469
        FMP   8003              0469   39  8003  0423
        STU  NUIIM              0423   21  0078  0531
        RAU  SNAAX              0531   60  0370  0275
        FMP  SHAAX              0275   39  0320  0470
        STU  FSX                0470   21  0034  0087
        FMP   8003              0087   39  8003  0391
        FAD  NUIIM              0391   32  0078  0305
        FDV  DENNM              0305   34  0174  0274
        LDD  ABCD1  EOOAU       0274   69  0503  0506
COSHX   STD  NEXTC              0169   24  0022  0325
        STU  ARG                0325   21  0280  0083
        LDD         EOOEA       0083   69  0086  0050
        STU  EARGP              0086   21  0040  0193
        RSU  ARG                0193   61  0280  0035
        LDD         EOOEA       0035   69  0638  0050
        FAD  EARGP              0638   32  0040  0017
        FDV  TWO    NEXTC       0017   34  0520  0022
SINHX   STD  NEXTS              0319   24  0072  0375
        STU  ARG                0375   21  0280  0183
        RSU  ARG                0183   61  0280  0085
        LDD         EOOEA       0085   69  0688  0050
        STU  EARGM              0688   21  0492  0545
        RAU  ARG                0545   60  0280  0185
        LDD         EOOEA       0185   69  0738  0050
        FSB  EARGM              0738   33  0492  0519
        FDV  TWO    NEXTS       0519   34  0520  0072
EOOCR   STD  EXIT               0269   24  0172  0425
        BMI  NEGAT  REDUC       0425   46  0178  0179
NEGAT   FAD  TWOPI              0178   32  0631  0357
        BMI  NEGAT              0357   46  0178  0061
        FSB  ONEPI  COSIO       0061   33  0014  0441
REDUC   FSB  TWOPI              0179   33  0631  0407
        BMI         REDUC       0407   46  0360  0179
        FAD  ONEPI  COSIO       0360   32  0014  0441
COSIO   STU  THETA              0441   21  0196  0199
        RSU  FPONE              0199   61  0302  0457
        STU  TERMM              0457   21  0462  0165
        STU  FUNKT              0165   21  0570  0473
        STL  ENN    NEGST       0473   20  0427  0330
EOOSR   STD  EXIT               0369   24  0172  0475
        BMI  NEGAV  REDUU       0475   46  0328  0279
NEGAV   FAD  TWOPI              0328   32  0631  0507
        BMI  NEGAV              0507   46  0328  0161
        FSB  ONEPI  SINET       0161   33  0014  0491
REDUO   FSB  TWOPI              0279   33  0631  0557
        BMI         REOUU       0557   46  0410  0279
        FAO  ONEPI  SINET       0410   32  0014  0491
SINET   STU  THETA              0491   21  0196  0299
        RSU   8003              0299   61  8003  0607
        STU  TERMM              0607   21  0462  0265
        STU  FUNKT              0265   21  0570  0523
        LDD  FPONE              0523   69  0302  0355
        STD  ENN    NEGST       0355   24  0427  0330
NEGST   RAU  ENN                0330   60  0427  0681
        FAD  FPONE              0681   32  0302  0329
        STU  NPONE              0329   21  0484  0187
        FAD  FPONE              0187   32  0302  0379
        STU  ENN                0379   21  0427  0380
        RSU  TERMM              0380   61  0462  0067
        FMP  THETA              0067   39  0196  0296
        FMP  THETA              0296   39  0196  0346
        FDV  NPONE              0346   34  0484  0534
        FDV  ENN                0534   34  0427  0477
        STU  TERMM              0477   21  0462  0315
        RAM  FUNKT              0315   67  0570  0525
        STL  FMAG               0525   20  0429  0032
        RAM  TERMM              0032   67  0462  0167
        RAU   8002              0167   60  8002  0575
        FDV  FMAG               0575   34  0429  0479
        FSB  SIZES              0479   33  0082  0509
        BMI  ENUFF              0509   46  0512  0513
        RAU  FUNKT              0513   60  0570  0625
        FAD  TERMM              0625   32  0462  0189
        STU  FUNKT  NEGST       0189   21  0570  0330
ENUFF   RAU  FUNKT  EXIT        0512   60  0570  0172
SIZES    10   0000   0043       0082   10  0000  0043
TWOPI    62   8318   5351       0631   62  8318  5351
ONEPI    31   4159   2751       0014   31  4159  2751
FPONE    10   0000   0051       0302   10  0000  0051
EOOAU   STD  SEXT               0506   24  0559  0562
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         BMI  SERR                   237   0562   46 0365 0516
         NZE            SEXT         238   0516   45 0620 0559
         STU  SA                     239   0620   21 0374 0527
         FAD  S10                    240   0527   32 0430 0657
         FMP  SHAF      SB           241   0657   39 0460 0510
SB       STU  SSAV      SAB          242   0510   21 0064 0267
SAB      RAU  SA                     243   0267   60 0374 0529
         FDV  SSAV                   244   0529   34 0064 0164
         FAD  SSAV                   245   0164   32 0064 0541
         FMP  SHAF                   246   0541   39 0460 0560
         FSB  SSAV                   247   0560   33 0064 0591
         NZU            SR           248   0591   44 0595 0396
         BMI            SR           249   0595   46 0098 0396
         FAD  SSAV                   250   0098   32 0064 0641
         STU  SSAV      SAB          251   0641   21 0064 0267
SR       RAU  SSAV      SEXT         252   0396   60 0064 0559
SERR     HLT   0000     SEXT         253   0365   01 0000 0559
SHAF      50   0000      0050        254   0460   50 0000 0050
S10       10   0000      0051        255   0430   10 0000 0051
START    RCD   1951                  256   1999   70 1951 0401
         LDD   1951                  257   0401   69 1951 0304
         STD  X                      258   0304   24 0707 0610
         LDD   1952                  259   0610   69 1952 0405
         STD  T                      260   0405   24 0058 0261
         LDD  ZERO                   261   0261   69 0264 0317
         STD  HOLD                   262   0317   24 0670 0573
         LDD  INDXM                  263   0573   69 0076 0579
         RAB   8001                  264   0579   82 8001 0285
         LDD  ZERO                   265   0285   69 0264 0367
         STD  HOLD      LOPP1        266   0367   24 0670 0623
LOPP1    RAU  STAA   B               267   0623   60 4120 0675
         FDV  ALPHA                  268   0675   34 0278 0378
         FDV  TWO                    269   0378   34 0520 0720
         LDD            EOOAU        270   0720   69 0673 0506
         STU  SQRRT                  271   0673   21 0428 0731
         FMP  X                      272   0731   39 0707 0757
         STU  ARGGX                  273   0757   21 0459 0612
         RAU  SQRRT                  274   0612   60 0428 0283
         FMP  L                      275   0283   39 0186 0286
         STU  ARGGL                  276   0286   21 0176 0629
         LDD            ZMXSB        277   0629   69 0182 0450
         STU  ZMX                    278   0182   21 0336 0289
         LDD            PHISB        279   0289   69 0542 0000
         RAU  STAA   B               280   0542   60 4120 0725
         FMP  T                      281   0725   39 0058 0308
         FAD  THETO                  282   0308   32 0010 0287
         LDD            EOOCR        283   0287   69 0090 0269
         FMP  ZMX                    284   0090   39 0336 0386
         FMP  AMMM   B               285   0386   39 4100 0500
         STU  A1     B               286   0500   21 4140 0293
         SXB   0001                  287   0293   53 0001 0349
         NZB  LOPP1     CONT1        288   0349   42 0623 0553
CONT1    LDD  NOLAM                  289   0553   69 0456 0609
         RAB   8001     LOOP9        290   0609   82 8001 0415
LOOP9    RAU  ALPHA                  291   0415   60 0278 0333
         FMP  KAPPA                  292   0333   39 0436 0486
         FMP  KAPPA                  293   0486   39 0436 0536
         STU  ALKAP                  294   0536   21 0190 0343
         FSB  LAMDA  B               295   0343   33 4220 0497
         STU  DIFF                   296   0497   21 0352 0455
         RAU  LAMDA  B               297   0455   60 4220 0775
         FMP  T                      298   0775   39 0058 0358
         LDD            EOOEA        299   0358   69 0311 0050
         STU  ELAMT                  300   0311   21 0566 0569
         RAU  ELAMT                  301   0569   60 0566 0071
         FMP  QQQ                    302   0071   39 0324 0424
         FMP  AYEJ   B               303   0424   39 4200 0550
         FMP  ALPHA                  304   0550   39 0278 0478
         FDV  KAY                    305   0478   34 0581 0781
         FDV  DIFF                   306   0781   34 0352 0402
         STU  A3P                    307   0402   21 0556 0659
         RAU  KAPPA                  308   0659   60 0436 0691
         FMP  X                      309   0691   39 0707 0807
         LDD            COSHX        310   0807   69 0660 0169
         STU  CSHKX                  311   0660   21 0314 0417
         RAU  KAPPA                  312   0417   60 0436 0741
         FMP  L                      313   0741   39 0186 0586
         LDD            COSHX        314   0586   69 0339 0169
         STU  CSHKL                  315   0339   21 0044 0547
         RAU  LAMDA  B               316   0547   60 4220 0825
         BMI  NEG       POS          317   0825   46 0528 0679
NEG      RSU  LAMDA  B               318   0528   61 4220 0875
         FDV  ALPHA                  319   0875   34 0278 0578
         LDD            EOOAU        320   0578   69 0831 0506
         STU  SQRLA                  321   0831   21 0636 0389
         FMP  X                      322   0389   39 0707 0857
         LDD            EOOCR        323   0857   69 0710 0269
         STU  COSLX                  324   0710   21 0364 0467
         RAU  SQRLA                  325   0467   60 0636 0791
         FMP  L                      326   0791   39 0186 0686
         LDD            EOOCR        327   0686   69 0439 0269
         STU  COSLL                  328   0439   21 0094 0597
         RAU  COSLX                  329   0597   60 0364 0619
         FDV  COSLL                  330   0619   34 0094 0194
         FMP  CSHKL                  331   0194   39 0044 0294
         FSB  CSHKX                  332   0294   33 0314 0841
         FMP  A3P                    333   0841   39 0556 0606
         STU  A3     B               334   0606   21 4240 0393
         SXB   0001                  335   0393   53 0001 0399
         NZB  LOOP9     CONT3        336   0399   42 0415 0603
POS      FDV  ALPHA                  337   0679   34 0278 0628
         LDD            EOOAU        338   0628   69 0881 0506
         STU  SQRLA                  339   0881   21 0636 0489
         FMP  X                      340   0489   39 0707 0907
         LDD            COSHX        341   0907   69 0760 0169
         STU  CSHLX                  342   0760   21 0414 0517
         RAU  SQRLA                  343   0517   60 0636 0891
         FMP  L                      344   0891   39 0186 0736
         LDD            COSHX        345   0736   69 0539 0169
         STU  CSHLL                  346   0539   21 0344 0647
         RAU  CSHLX                  347   0647   60 0414 0669
         FDV  CSHLL                  348   0669   34 0344 0394
         FMP  CSHKL                  349   0394   39 0044 0444
         FSB  CSHKX                  350   0444   33 0314 0941
         FMP  A3P                    351   0941   39 0556 0656
         STU  A3     B               352   0656   21 4240 0443
         SXB   0001                  353   0443   53 0001 0449
         NZB  LOOP9     CONT3        354   0449   42 0415 0603
CONT3    RAA   0001                  355   0603   80 0001 0709
         RAU  ONE                    356   0709   60 0662 0567
         RSL   8003                  357   0567   66 8003 0925
         STL  N                      358   0925   20 0729 0282
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          RAU  ZERO
          STU  HOLO        LOOP3
LOOP3     RAU  N
          FAD  TWO
          STU  N
          FMP  PI
          FDV  TWO
          FDV  L
          STU  ARG1
          FMP  ARG1
          FMP  ALPHA
          STU  ARG2
          RAU  ZERO
          STU  A24
          LDD  NDLAM
          RAB   8001       LOP5
LOP5      RAU  ARG2
          FAD  LAMDA  B
          STU  A2S41
          RAU  ARG2
          FAD  ALKAP
          STU  A2S42
          RAU  CSHKL
          FMP  ALPHA
          FMP  SOU
          FMP  AYEJ   B
          FDV  KAY
          FDV  A2S41
          FDV  A2S42
          FAD  A24
          STU  A24
          SXB   0001
          NZB  LOP5        CONT9
CONT9     RAU  ZERO
          STD  SETT
          LDD  INDXM
          RAB   8001       LUPP2
LOPP2     RAU  ARG2
          FMP  ARG2
          STU  ARGSQ
          RAU  STAA   B
          FMP  STAA   B
          FAD  ARGSQ
          STU  DENOM
          RAU  AMMM   B
          FMP  ARG2
          FDV  DENOM
          FAD  SETT
          STU  SETT
          SXB   0001
          NZB  LOPP2       CONT4
CONT4     FAD  A24
          STU  A2SUM
          RAU  ARG2
          FMP  T
          FSB  FIFTY
          BMI  GD          RTO
STO       LDD  ZERO
          STD  TERM
          RAU  TERM
          FAD  HOLO
          STU  HOLO
          NZU              CONT5
          RAU  HOLO
          FSB  TERM        MOREC
GO        RSU  ARG2,
          FMP  T
          LDD              EOOEA
          STU  EAG2T
          RAU  ARG1
          FMP  X
          LDD              EOOCR
          STU  COS1X
          RAU  L
          FMP  L
          FDV  ALPHA
          FDV  N
          FDV  PI
          STU  A2DIV
          RAU  EAG2T
          FMP  COS1X
          FDV  A2DIV
          STU  A2DOT
          FMP  A2SUM
          STU  TERM        MOREC
MOREC     BMA  SUB         ADD
SUB       RAA   0001
          RAU  HOLO
          FSB  TERM
          STU  HOLO
          RAU  TERM
          FDV  HOLO
          RAM   8003
          RAU   8002
          FSB  CRIT
          BMI  CONT5       LOOP3
ADD       RSA   0001
          RAU  HOLO
          FAD  TERM
          STU  HOLO
          RAU  TERM
          FDV  HOLO
          RAM   8003
          RAU   8002
          FSB  CRIT
          BMI  CONT5       LOOP3
CONT5     LDD  INDXM
          RAB   8001
          RAU  ZERO
          STU  CELL        LOP
LOP       RAU  A1     B
          FAD  CELL
          STU  CELL
          SXB   0001
          NZB  LOP         GOON1
GOON1     LDD  NDLAM
          RAB   8001
          RAU  ZERO
          STU  CELL3       LOP2
LOP2      RAU  A3     B
          FAD  CELL3
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359   0282   60 0264 0719
360   0719   21 0670 0723
361   0723   60 0729 0383
362   0383   32 0520 0697
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364   0332   39 0018 0168
365   0168   34 0520 0770
366   0770   34 0186 0786
367   0786   21 0290 0493
368   0493   39 0290 0340
369   0340   39 0278 0678
370   0678   21 0382 0335
371   0335   60 0264 0769
372   0769   21 0474 0577
373   0577   69 0456 0759
374   0759   82 8001 0465
375   0465   60 0382 0337
376   0337   32 4220 0747
377   0747   21 0452 0505
378   0505   60 0382 0387
379   0387   32 0190 0617
380   0617   21 0272 0975
381   0975   60 0044 0499
382   0499   39 0278 0728
383   0728   39 0324 0524
384   0524   39 4200 0600
385   0600   34 0581 0931
386   0931   34 0452 0502
387   0502   34 0272 0322
388   0322   32 0474 0451
389   0451   21 0474 0627
390   0627   53 0001 0433
391   0433   42 0465 0437
392   0437   60 0264 0819
393   0819   24 0372 1025
394   1025   69 0076 0779
395   0779   82 8001 0385
396   0385   60 0382 0487
397   0487   39 0382 0432
398   0432   21 0836 0589
399   0589   60 4120 1075
400   1075   39 4120 0820
401   0820   32 0836 0563
402   0563   21 0268 0171
403   0171   60 4100 0555
404   0555   39 0382 0482
405   0482   34 0268 0318
406   0318   32 0372 0549
407   0549   21 0372 1125
408   1125   53 0001 0981
409   0981   42 0385 0435
410   0435   32 0474 0501
411   0501   21 0706 0809
412   0809   60 0382 0537
413   0537   39 0058 0408
414   0408   33 0361 0587
415   0587   46 0390 0991
416   0991   69 0264 0667
417   0667   24 0870 0773
418   0773   60 0870 1175
419   1175   32 0670 0797
420   0797   21 0670 0823
421   0823   44 0677 0778
422   0677   60 0670 1225
423   1225   33 0870 0847
424   0390   61 0382 0637
425   0637   39 0058 0458
426   0458   69 0411 0050
427   0411   21 0616 0869
428   0869   60 0290 0645
429   0645   39 0707 0957
430   0957   69 0810 0269
431   0810   21 0464 0717
432   0717   60 0186 1041
433   1041   39 0186 0886
434   0886   34 0278 0928
435   0828   34 0729 0829
436   0829   34 0018 0368
437   0368   21 0422 1275
438   1275   60 0616 0271
439   0271   39 0464 0514
440   0514   34 0422 0472
441   0472   21 0276 0879
442   0879   39 0706 0756
443   0756   21 0870 0847
444   0847   41 0650 0551
445   0650   80 0001 0806
446   0806   60 0670 1325
447   1325   33 0870 0897
448   0897   21 0670 0873
449   0873   60 0870 1375
450   1375   34 0670 0920
451   0920   67 8003 0727
452   0727   60 8002 0485
453   0485   33 0788 0515
454   0515   46 0778 0723
455   0551   81 0001 1007
456   1007   60 0670 1425
457   1425   32 0870 0947
458   0947   21 0670 0923
459   0923   60 0870 1475
460   1475   34 0670 0970
461   0970   67 8003 0777
462   0777   60 8002 0535
463   0535   33 0788 0565
464   0565   46 0778 0723
465   0778   69 0076 0929
466   0929   82 8001 0585
467   0585   60 0264 0919
468   0919   21 0574 0827
469   0827   60 4140 0695
470   0695   32 0574 0601
471   0601   21 0574 0877
472   0877   53 0001 0483
473   0483   42 0827 0687
474   0687   69 0456 0859
475   0859   82 8001 0615
476   0615   60 0264 0969
477   0969   21 0624 0927
478   0927   60 4240 0745
479   0745   32 0624 0651
```

```
          STU  CELL3                    480   0651   21 0624 0777
          SXB   0001                    481   0977   53 0001 0533
          NZB  LOP2      GOON8          482   0533   42 0927 0737
GOON8     RSU  HOLD                     483   0737   61 0670 1525
          FAD  CELL                     484   1525   32 0574 0701
          FAD  CELL3                    485   0701   32 0624 0751
          STU  ANSWR     FINIS          486   0751   21 0856 0909
FINIS     LDD  ANSWR                    487   0909   69 0856 0959
          STD   1977                    488   0959   24 1977 0480
          LDD  CELL                     489   0480   69 0574 1027
          STD   1978                    490   1027   24 1978 1031
          LDD  HOLD                     491   1031   69 0670 0973
          STD   1979                    492   0973   24 1979 0532
          LDD  CELL3                    493   0532   69 0624 1077
          STD   1980                    494   1077   24 1980 0583
          LDD  X                        495   0583   69 0707 0860
          STD   1981                    496   0860   24 1981 0584
          LDD  T                        497   0584   69 0058 0461
          STD   1982                    498   0461   24 1982 0635
          PCH   1977     START          499   0635   71 1977 1999
ONE        10   0000      0051          500   0662   10 0000 0051
TWO        20   0000      0051          501   0520   20 0000 0051
PI         31   4159      2751          502   0018   31 4159 2751
FIFTY      50   0000      0052          503   0361   50 0000 0052
ZERO       00   0000      0000          504   0264   00 0000 0000
CRIT       10   0000      0047          505   0788   10 0000 0047
ARTAN     STD  EXIT                       4   1800   24 1803 1806
          NZE            EXIT             5   1806   45 1810 1803
          SMI  MINUS                      6   1810   46 1813 1814
          STU  ARTAO                      7   1814   21 1818 1821
          LDD  FPONE                      8   1821   69 1824 1827
          STD  ENNNN                      9   1827   24 1830 1833
          STD  AYE       SUBTR           10   1833   24 1836 1839
MINUS     RSL   8003                     11   1813   66 8003 1871
          STL  ARTAO                     12   1871   20 1818 1822
          RSU  FPONE                     13   1822   61 1824 1829
          STD  ENNNN                     14   1829   24 1830 1883
          STU  AYE       SUSTR           15   1883   21 1836 1839
SUSTR     RAU  ARTAO                     16   1839   60 1818 1823
          FSB  FPONE                     17   1823   33 1824 1801
          NZE  DIFFE                     18   1801   45 1804 1805
          LDD  PIOV4                     19   1805   69 1808 1811
          STD  FUNGT     MULTA           20   1811   24 1864 1817
DIFFE     SMI  SMALL                     21   1804   46 1807 1858
          FSB  FPONE                     22   1858   33 1824 1851
          BMI  NEGAT     POSIT           23   1851   46 1854 1855
NEGAT     LDD  PIOV4                     24   1854   69 1808 1861
          STD  FUNGT                     25   1861   24 1864 1867
          RAU  ARTAO                     26   1867   60 1818 1873
          FAD  FPONE                     27   1873   32 1824 1802
          STU  TURRN                     28   1802   21 1856 1809
          FSB  FPTWO                     29   1809   33 1812 1889
          FDV  TURRN     COMBI           30   1889   34 1856 1857
SMALL     FAD  ARTAO                     31   1807   32 1818 1845
          SMI            NEGAT           32   1845   46 1848 1854
          RAU  ARTAO                     33   1848   60 1818 1874
          FSB  LOBND                     34   1874   33 1877 1853
          BMI  TINEY                     35   1853   46 1859 1860
          RAU  ARTAO                     36   1860   60 1818 1825
          STL  FUNGT     COMBI           37   1825   20 1864 1857
POSIT     LDD  PIOV2                     38   1855   69 1862 1815
          STD  FUNGT                     39   1815   24 1864 1868
          FSB  UPBND                     40   1868   33 1872 1849
          SMI            MULTA           41   1849   46 1852 1817
          RSU  FPONE                     42   1852   61 1824 1879
          FDV  ARTAO     COMBI           43   1879   34 1818 1857
COMBI     STU  TURRN                     44   1857   21 1856 1863
          STD  TURRM                     45   1863   24 1816 1819
          FMP  TURRN                     46   1819   39 1856 1865
          STU  ARGUE     FIGUR           47   1865   21 1820 1875
FIGUR     RAU  TURRM                     48   1875   60 1816 1826
          FAD  FUNGT                     49   1826   32 1864 1841
          STU  FUNGT                     50   1841   21 1864 1869
          RAU  ENNNN                     51   1869   60 1830 1835
          FAD  FPTWO                     52   1835   32 1812 1840
          STU  ENNNN                     53   1840   21 1830 1834
          RSU  TURRN                     54   1834   61 1856 1866
          FMP  ARGUE                     55   1866   39 1820 1870
          STU  TURRN                     56   1870   21 1856 1876
          FDV  ENNNN                     57   1876   34 1830 1880
          STU  TURRM                     58   1880   21 1816 1828
          RAM  FUNGT                     59   1828   67 1864 1878
          STL  FMAGG                     60   1878   20 1884 1837
          RAM  TURRM                     61   1837   67 1816 1831
          RAU   8002                     62   1831   60 8002 1890
          FDV  FMAGG                     63   1890   34 1884 1885
          FSB  SIZEB                     64   1885   33 1838 1881
          SMI  MULTA     FIGUR           65   1881   46 1817 1875
TINEY     RAU  ARTAO     MULTA           66   1859   60 1818 1817
MULTA     RAU  AYE                       67   1817   60 1836 1891
          FMP  FUNGT     EXIT            68   1891   39 1864 1803
FPONE      10   0000      0051           69   1824   10 0000 0051
FPTWO      20   0000      0051           70   1812   20 0000 0051
SIZEB      10   0000      0043           71   1838   10 0000 0043
PIOV2      15   7079      6351           72   1862   15 7079 6351
PIOV4      78   5398      1650           73   1808   78 5398 1650
UPBND      10   0000      0060           74   1872   10 0000 0060
LOBND      10   0000      0040           75   1877   10 0000 0040
```

APPENDIX E

Description and Explanation of the IBM-650 Computer Program Used to Calculate Surface Heat Flow.

The computer program was written to calculate the heat flow out of the fuel and into the moderator. The heat flow out of the fuel surface is given by

$$
\begin{aligned}
(q/A)_f(t) = -k_f \Bigg\{ & \sum_{i=1}^{p} \sqrt{\frac{\beta_i}{2\alpha}}\; B_i \left(D_i \cos \beta_i t + E_i \sin \beta_i t\right) \\
& + \sum_{n=1,3,5,\cdots}^{\infty} \frac{\left(\frac{n}{2L}\right) e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{\frac{L^2}{n\pi\alpha}} \left(\sum_{i=1}^{p} \frac{B_i \left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2} \right. \\
& \qquad \left. + \sum_{j=1}^{s} \frac{q_{oo}\ \alpha\ A_j \cosh \kappa L}{k_m \left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\kappa^2\right)} \right) \\
& + \sum_{j=1}^{s} \frac{q_{oo} - A_j\, e^{\lambda_j t}}{k_f\left(\alpha\kappa^2 - \lambda_j\right)} \left[\frac{\sqrt{\frac{\lambda_j}{\alpha}} \cosh(\kappa L) \sinh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)}{\cosh\left(\sqrt{\frac{\lambda_j}{\alpha}}\, L\right)} - \kappa \sinh \kappa L \right] \Bigg\}
\end{aligned}
\qquad \text{(E-1)}
$$

and the heat flow into the moderator is given by

$$
\begin{aligned}
(q/A)_m(t) = \rho\, k_m \Bigg\{ & \sum_{i=1}^{p} \sqrt{\frac{\beta_i}{2\alpha}}\; B_i \left(D_i \cos \beta_i t + E_i \sin \beta_i t\right) \\
& + \sum_{n=1,3,5,\cdots}^{\infty} \frac{\left(\frac{n\pi}{2L}\right) e^{-\frac{n^2\pi^2\alpha}{4L^2} t}}{\frac{L^2}{n\pi\alpha}} \left(\sum_{i=1}^{p} \frac{B_i \left(\frac{n^2\pi^2\alpha}{4L^2}\right)}{\frac{n^4\pi^4\alpha^2}{16L^4} + \beta_i^2} + \sum_{j=1}^{s} \frac{q_{oo}\ \alpha\ A_j \cosh \kappa L}{k_m \left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2}\right)} \right)
\end{aligned}
$$

$$-\sum_{j=1}^{s} \frac{\alpha F A_j}{k_m \lambda_j} \left(\frac{\sqrt{\frac{\lambda_j}{\alpha}} \sinh \sqrt{\frac{\lambda_j}{\alpha}} L}{\cosh \sqrt{\frac{\lambda_j}{\alpha}} L} \right) ,$$

(E-2)

$$\text{where } D_i = \frac{\cosh \gamma_i L \ \sinh \gamma_i L - \cos \gamma_i L \ \sin \gamma_i L}{\cos^2 \gamma_i L \ \cosh^2 \gamma_i L + \sin^2 \gamma_i L \ \sinh^2 \gamma_i L} ,$$

$$E_i = \frac{\cosh \gamma_i L \ \sinh \gamma_i L + \cos \gamma_i L \ \sin \gamma_i L}{\cos^2 \gamma_i L \ \cosh^2 \gamma_i L + \sin^2 \gamma_i L \ \sinh^2 \gamma_i L}$$

and $\gamma_i = \sqrt{\frac{\beta_i}{2\alpha}}$.

The equivalence between elements of the algebraic equation and the symbolic logic of the computer program is shown in Table E-1.

Table E-1. Definition of symbolic terms of the IBM-650 computer program for calculating surface heat flow.

$$\gamma_i = \sqrt{\frac{\beta_i}{2\alpha}}$$

$$ZMX1_i = \frac{\cosh \gamma_i L \ \sinh \gamma_i L - \cos \gamma_i L \ \sin \gamma_i L}{\cos^2 \gamma_i L \ \cosh^2 \gamma_i L + \sin^2 \gamma_i L \ \sinh^2 \gamma_i L} = D_i$$

$$A1_i = \gamma_i A_i \ (ZMX1_i \cos \beta_i t - ZMX2_i \sin \beta_i t)$$

$$A3P_j = \frac{q_\infty \alpha A_j e^{\lambda_j t}}{k(\alpha\kappa^2 - \lambda_j)}$$

$$A3SUM_j = \frac{\sqrt{\frac{\lambda_j}{\alpha}} \cosh \kappa L \ \sinh \sqrt{\frac{\lambda_j}{\alpha}} L}{\cosh \sqrt{\frac{\lambda_j}{\alpha}} L} - \kappa \sinh \kappa L$$

$$ZMX2_i = \frac{\cos \gamma_i L \ \sin \gamma_i L + \cosh \gamma_i L \ \sinh \gamma_i L}{\cos^2 \gamma_i L \ \cosh^2 \gamma_i L + \sin^2 \gamma_i L \ \sinh^2 \gamma_i L} = E_i$$

Table E-1 cont.

$$A3_j = (A3P_j)\,(A3SUM_j)$$

$$CSHLL_j = \cosh \sqrt{\frac{\lambda_j}{\alpha}}\, L$$

$$COSLL_j = \cos \sqrt{\frac{\lambda_j}{\alpha}}\, L$$

$$ARG1_n = n\pi/2L$$

$$ARG2_n = n^2\pi^2\alpha/4L^2$$

$$A2ST1_i = B_i\left(\frac{n^2\pi^2\alpha}{4L^2}\right) / \left(\frac{n^4\pi^4\alpha^2}{16L^2} + \beta_i^2\right)$$

$$A2ST1_j = q_\infty\, \alpha\, A_j \cosh \kappa L \;/\; k\left(\frac{n^2\pi^2\alpha}{4L^2} + \lambda_j\right)\left(\frac{n^2\pi^2\alpha}{4L^2} + \alpha\,\kappa^2\right)$$

$$A2DDT_n = \frac{\frac{n\pi}{2L}\, e^{-\frac{n^2\pi^2\pi}{4L^2}t}}{L^2 / n\pi\alpha}$$

$$A2SUM_n = \sum_{i=1}^{p} A2ST1_i + \sum_{j=1}^{s} A2ST1_j$$

$$TERM_n = (A2DDT_n)\,(A2SUM_n)$$

The input data consists of the heat generation and surface temperature parameters, appropriate material constants, half-thickness of the region and numerical constants. Table E-2 lists input data required for the program.

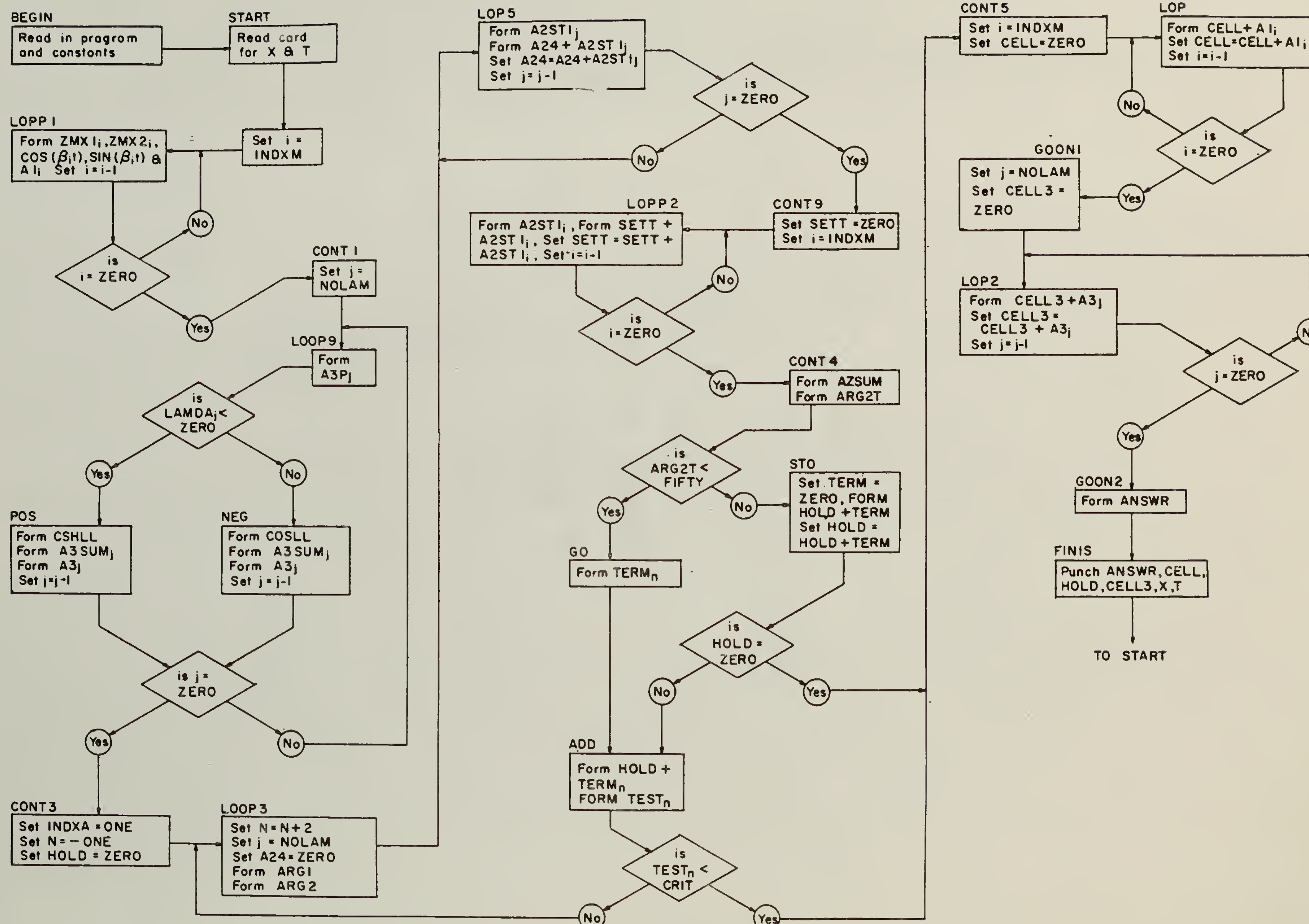
Table E-2. Input Data Required for Use of the IBM-650 Computer Program Used to Calculate Surface Heat Flow Rates.

Symbol	Explanation	Storage Location
ZERO	0.00	0164
ONE	1.00	0656
TWO	2.00	0820
Pi	3.14159	0779
FIFTY	50.00	0461
CRIT	0.0001	0388
ALPHA	Thermal Diffusivity	0278
KAPPA	Reciprocal of Thermal Neutron Diffusion Length in Fuel	0436
Q00	Normalization Factor for Heat Generation	0324
KAY	Thermal Conductivity	0581
L	Half-thickness of Region	0186
INDXM	No. of Terms, Surface Temperature Fit (00000000xx)	0076
NOLAM	No. of Terms, Heat Generation Fit (00000000xx)	0456
$AYEJ_j$	Amplitude Parameter, Heat Generation Fit	(0200 + j)
$LAMDA_j$	Exponential Parameter, Heat Generation Fit	(0020 + j)
$AMMM_j$	Amplitude Parameter, Surface Temperature Fit	(0100 + j)
$BTAA_j$	Period Parameter, Surface Temperature Fit	(0120 + j)

The output from this program is punched out on one card having an eight word capacity, one word consisting of 10 digits and a sign. The form of the output is shown in Table E-3.

Table E-3. Output form for IBM-650 Computer Program Used to Calculate Surface Heat Flow Rates.

WORD 1	WORD 2	WORD 3	WORD 4	WORD 5	WORD 6	WORD 7	WORD 8
(q/A)	$\sum_{i=1}^{p} A1_i$	$\sum_{n=1,3,5}^{\infty} (\text{Term})_n$	$\sum_{j=1}^{s} A3_j$	--	t	--	--



LOGIC DIAGRAM-APPENDIX E

OBJECT PROGRAM-APPENDIX E

	BLR	0200	0260	1	0000	00	0000	0000
	BLR	1800	1899	2	0000	00	0000	0000
	SYN	ARTAN	1800	3	0000	00	0000	0000
	SYN	START	1999	4	0000	00	0000	0000
	SYN	AYEJ	0200	5	0000	00	0000	0000
	SYN	LAMDA	0220	6	0000	00	0000	0000
	SYN	A3	0240	7	0000	00	0000	0000
	BLR	1951	1960	8	0000	00	0000	0000
	BLR	1977	1984	9	0000	00	0000	0000
	BLR	0100	0160	10	0000	00	0000	0000
	SYN	AMMM	0100	11	0000	00	0000	0000
	SYN	STAA	0120	12	0000	00	0000	0000
	SYN	ALPHA	0278	13	0000	00	0000	0000
	SYN	KAPPA	0436	14	0000	00	0000	0000
	SYN	L	0186	15	0000	00	0000	0000
	SYN	QOO	0324	16	0000	00	0000	0000
	SYN	KAY	0581	17	0000	00	0000	0000
	SYN	INOXM	0076	18	0000	00	0000	0000
	SYN	NOLAM	0456	19	0000	00	0000	0000
	SYN	A1	0140	20	0000	00	0000	0000
EOOEA	STO	AAA1		21	0000	24	0003	0006
	STU	AAA14		22	0006	21	0010	0013
	RAU	AAA16		23	0013	60	0016	0021
	FAM	AAA14		24	0021	37	0010	0037
	STU	AAA2		25	0037	21	0042	0045
	LOD	AAA3		26	0045	69	0048	0001
	STD	AAA4	AAA8	27	0001	24	0004	0007
AAA8	RAU	AAA2		28	0007	60	0042	0047
	FSB	AAA5		29	0047	33	0050	0027
	BMI	AAA6		30	0027	46	0030	0031
	STU	AAA2		31	0031	21	0042	0095
	RAU	AAA4		32	0095	60	0004	0009
	FMP	AAA7		33	0009	39	0012	0062
	STU	AAA4	AAA8	34	0062	21	0004	0007
AAA6	RAU	AAA2		35	0030	60	0042	0097
	FSB	AAA3		36	0097	33	0048	0025
	BMI	AAA28		37	0025	46	0028	0029
	STU	AAA2		38	0029	21	0042	0195
	RAU	AAA4		39	0195	60	0004	0059
	FMP	AAA9		40	0059	39	0162	0262
	STU	AAA4	AAA6	41	0262	21	0004	0030
AAA28	RAU	AAA2		42	0028	60	0042	0197
	FSB	AAA10		43	0197	33	0300	0077
	BMI	AAA11		44	0077	46	0080	0081
	STU	AAA2		45	0081	21	0042	0295
	RAU	AAA4		46	0295	60	0004	0309
	FMP	AAA12		47	0309	39	0312	0362
	STU	AAA4	AAA28	48	0362	21	0004	0028
AAA11	RAU	AAA2		49	0080	60	0042	0297
	LDD		AAA17	50	0297	69	0350	0053
	FMP	AAA4		51	0350	39	0004	0054
	STU	AAA13		52	0054	21	0008	0011
	RAU	AAA14		53	0011	60	0010	0015
	BMI	AAA15		54	0015	46	0018	0019
	RAU	AAA13	AAA1	55	0019	60	0008	0003
AAA15	RAU	AAA3		56	0018	60	0048	0303
	FDV	AAA13	AAA1	57	0303	34	0008	0003
AAA17	STO	AAA18		58	0053	24	0056	0359
	RAU	AAA3		59	0359	60	0048	0353
	FAO	AAA2		60	0353	32	0042	0069
	STU	AAA19		61	0069	21	0024	0177
	LOD	AAA27		62	0177	69	0180	0033
	STO	AAA20		63	0033	24	0036	0039
	STO	AAA21		64	0039	24	0092	0345
	RAU	AAA2		65	0345	60	0042	0347
	FMP	AAA2		66	0347	39	0042	0192
	STU	AAA23	AAA22	67	0192	21	0046	0049
AAA22	FDV	AAA21		68	0049	34	0092	0292
	STU	AAA24		69	0292	21	0096	0099
	FAO	AAA19		70	0099	32	0024	0051
	STU	AAA19		71	0051	21	0024	0277
	RAU	AAA24		72	0277	60	0096	0301
	FOV	AAA19		73	0301	34	0024	0074
	FSB	AAA25		74	0074	33	0327	0403
	BMI		AAA26	75	0403	46	0306	0057
	RAU	AAA19	AAA18	76	0306	60	0024	0056
AAA26	RAU	AAA20		77	0057	60	0036	0041
	FAD	AAA3		78	0041	32	0048	0075
	STU	AAA20		79	0075	21	0036	0089
	FMP	AAA21		80	0089	39	0092	0342
	STU	AAA21		81	0342	21	0092	0395
	RAU	AAA23		82	0395	60	0046	0351
	FMP	AAA2		83	0351	39	0042	0392
	STU	AAA23	AAA22	84	0392	21	0046	0049
AAA3	10	0000	0051	85	0048	10	0000	0051
AAA5	50	0000	0051	86	0050	50	0000	0051
AAA7	14	8410	0053	87	0012	14	8410	0053
AAA9	27	1830	0051	88	0162	27	1830	0051
AAA10	20	0000	0050	89	0300	20	0000	0050
AAA12	12	2140	0051	90	0312	12	2140	0051
AAA16	00	0000	0000	91	0016	00	0000	0000
AAA25	70	0000	0047	92	0327	70	0000	0047
AAA27	20	0000	0051	93	0180	20	0000	0051
ZMXSB	STO	ABC01		94	0400	24	0453	0356
	RAU	ARGGL		95	0356	60	0409	0063
	LDO		COSHX	96	0063	69	0066	0169
	STU	CHAAL		97	0066	21	0020	0023
	RAU	ARGGL		98	0023	60	0409	0163
	LDO		EOOCH	99	0163	69	0166	0269
	STU	CSAAL		100	0166	21	0070	0073
	RAU	ARGGL		101	0073	60	0409	0263
	LOO		SINHX	102	0263	69	0266	0319
	STU	SHAAL		103	0266	21	0170	0173
	RAU	ARGGL		104	0173	60	0409	0313
	LDO		EOOSR	105	0313	69	0316	0369
	STU	SNAAL		106	0316	21	0270	0273
	RAU	CHAAL		107	0273	60	0020	0175
	FMP	CSAAL		108	0175	39	0070	0320
	STU	FCL		109	0320	21	0174	0377
	FMP	CHAAL		110	0377	39	0020	0370
	FMP	CSAAL		111	0370	39	0070	0420
	STU	DEIIN		112	0420	21	0274	0427
	RAU	SNAAL		113	0427	60	0270	0275
	FMP	SHAAL		114	0275	39	0170	0470
	STU	FSL		115	0470	21	0374	0477

	RAU	CSAAL		116	0477	60	0070	0325
	FMP	SHAAL		117	0325	39	0170	0520
	STU	CSSHL		118	0520	21	0424	0527
	RAU	CSAAL		119	0527	60	0070	0375
	FMP	SNAAL		120	0375	39	0270	0570
	STU	CSSNL		121	0570	21	0474	0577
	RAU	CHAAL		122	0577	60	0020	0425
	FMP	SHAAL		123	0425	39	0170	0620
	STU	CHSHL		124	0620	21	0524	0627
	RAU	FSL		125	0627	60	0374	0079
	FMP	8003		126	0079	39	8003	0083
	STU	D2		127	0083	21	0038	0091
	RAU	FCL		128	0091	60	0174	0179
	FMP	8003		129	0179	39	8003	0183
	FAD	D2		130	0183	32	0038	0065
	STU	DENN2		131	0065	21	0670	0323
	RAU	CHSHL		132	0323	60	0524	0279
	FSB	CSSNL		133	0279	33	0474	0401
	FDV	DENN2		134	0401	34	0670	0720
	STU	ZMX1		135	0720	21	0574	0677
	RAU	CSSNL		136	0677	60	0474	0329
	FAD	CHSHL		137	0329	32	0524	0451
	FDV	DENN2		138	0451	34	0670	0770
	STU	ZMX2	ABCD1	139	0770	21	0624	0453
COSHX	STD	NEXTC		140	0169	24	0022	0475
	STU	ARG		141	0475	21	0280	0283
	LDD		EOOEA	142	0283	69	0086	0000
	STU	EARGP		143	0086	21	0040	0043
	RSU	ARG		144	0043	61	0280	0035
	LDD		EOOEA	145	0035	69	0088	0000
	FAD	EARGP		146	0088	32	0040	0017
	FDV	TWO	NEXTC	147	0017	34	0820	0022
SINHX	STD	NEXTS		148	0319	24	0072	0525
	STU	ARG		149	0525	21	0280	0333
	RSU	ARG		150	0333	61	0280	0085
	LDD		EOOEA	151	0085	69	0188	0000
	STU	EARGM		152	0188	21	0442	0445
	RAU	ARG		153	0445	60	0280	0185
	LDD		EOOEA	154	0185	69	0288	0000
	FSB	EARGM		155	0288	33	0442	0419
	FDV	TWO	NEXTS	156	0419	34	0820	0072
EOOCR	STD	EXIT		157	0269	24	0172	0575
	BMI	NEGAT	REDUC	158	0575	46	0078	0379
NEGAT	FAD	TWOPI		159	0078	32	0181	0307
	BMI	NEGAT		160	0307	46	0078	0061
	FSB	ONEPI	COSIO	161	0061	33	0014	0191
REDUC	FSB	TWOPI		162	0379	33	0181	0357
	BMI		REDUC	163	0357	46	0060	0379
	FAD	ONEPI	COSIO	164	0060	32	0014	0191
COSIO	STU	THETA		165	0191	21	0196	0199
	RSU	FPONE		166	0199	61	0002	0407
	STU	TERMM		167	0407	21	0412	0165
	STU	FUNKT		168	0165	21	0870	0373
	STL	ENN	NEGBT	169	0373	20	0727	0330
EOOSR	STD	EXIT		170	0369	24	0172	0625
	BMI	NEGAV	REDUD	171	0625	46	0178	0429
NEGAV	FAD	TWOPI		172	0178	32	0181	0457
	BMI	NEGAV		173	0457	46	0178	0161
	FSB	ONEPI	SINET	174	0161	33	0014	0291
REDUD	FSB	TWOPI		175	0429	33	0181	0507
	BMI		REDUD	176	0507	46	0310	0429
	FAD	ONEPI	SINET	177	0310	32	0014	0291
SINET	STU	THETA		178	0291	21	0196	0299
	RSU	8003		179	0299	61	8003	0557
	STU	TERMM		180	0557	21	0412	0265
	STU	FUNKT		181	0265	21	0870	0423
	LDD	FPONE		182	0423	69	0002	0005
	STD	ENN	NEGBT	183	0005	24	0727	0330
NEGBT	RAU	ENN		184	0330	60	0727	0281
	FAD	FPONE		185	0281	32	0002	0479
	STU	NPONE		186	0479	21	0034	0087
	FAD	FPONE		187	0087	32	0002	0529
	STU	ENN		188	0529	21	0727	0380
	RSU	TERMM		189	0380	61	0412	0067
	FMP	THETA		190	0067	39	0196	0296
	FMP	THETA		191	0296	39	0196	0346
	FDV	NPONE		192	0346	34	0034	0084
	FDV	ENN		193	0084	34	0727	0777
	STU	TERMM		194	0777	21	0412	0315
	RAM	FUNKT		195	0315	67	0870	0675
	STL	FMAG		196	0675	20	0579	0032
	RAM	TERMM		197	0032	67	0412	0167
	RAU	8002		198	0167	60	8002	0725
	FDV	FMAG		199	0725	34	0579	0629
	FSB	SIZES		200	0629	33	0082	0459
	BMI	ENUFF		201	0459	46	0462	0363
	RAU	FUNKT		202	0363	60	0870	0775
	FAD	TERMM		203	0775	32	0412	0189
	STU	FUNKT	NEGBT	204	0189	21	0870	0330
ENUFF	RAU	FUNKT	EXIT	205	0462	60	0870	0172
SIZES	10	0000	0043	206	0082	10	0000	0043
TWOPI	62	8318	5351	207	0181	62	8318	5351
ONEPI	31	4159	2751	208	0014	31	4159	2751
FPONE	10	0000	0051	209	0002	10	0000	0051
EOOAU	STD	SEXT		210	0450	24	0503	0406
	BMI	SERR		211	0406	46	0509	0360
	NZE		SEXT	212	0360	45	0064	0503
	STU	SA		213	0064	21	0068	0071
	FAD	S10		214	0071	32	0674	0501
	FMP	SHAF	SS	215	0501	39	0304	0354
SS	STU	SSAV	SAB	216	0354	21	0058	0261
SAB	RAU	SA		217	0261	60	0068	0473
	FDV	SSAV		218	0473	34	0058	0308
	FAD	SSAV		219	0308	32	0058	0285
	FMP	SHAF		220	0285	39	0304	0404
	FSB	SSAV		221	0404	33	0058	0335
	NZU		SR	222	0335	44	0289	0090
	BMI		SR	223	0289	46	0492	0090
	FAD	SSAV		224	0492	32	0058	0385
	STU	SSAV	SAB	225	0385	21	0058	0261
SR	RAU	SSAV	SEXT	226	0090	60	0058	0503
SERR	HLT	0000	SEXT	227	0509	01	0000	0503
SHAF	50	0000	0050	228	0304	50	0000	0050
S10	10	0000	0051	229	0674	10	0000	0051
START	RCU	1951		230	1999	70	1951	0551

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          LDD  1952                  231  0551  69 1952 0055
          STU  T                     232  0055  24 0358 0311
          LDD  ZERO                  233  0311  69 0164 0267
          STD  HOLD                  234  0267  24 0920 0523
          LDD  INDXM                 235  0523  69 0076 0679
          RAB   8001                 236  0679  82 8001 0435
          LDU  ZERO                  237  0435  69 0164 0317
          STD  HOLD        LOPP1     238  0317  24 0920 0573
LOPP1     RAU  STAA   B              239  0573  60 4120 0925
          FDV  ALPHA                 240  0825  34 0278 0328
          FDV  TWO                   241  0328  34 0820 0970
          LDD              EOOAU     242  0970  69 0623 0450
          STU  SQRRT                 243  0623  21 0378 0331
          FMP  L                     244  0331  39 0186 0286
          STU  ARGGL                 245  0286  21 0409 0512
          LDD              ZMXSR     246  0512  69 0365 0400
          RAU  STAA   B              247  0365  60 4120 0875
          FMP  T                     248  0875  39 0358 0408
          STU  PHEEE                 249  0408  21 0562 0415
          LDD              EODCR     250  0415  69 0168 0269
          FMP  ZMX1                  251  0168  39 0574 0724
          STU  A11                   252  0724  21 0428 0381
          RAU  PHEEE                 253  0381  60 0562 0367
          LDD              EOOSR     254  0367  69 1020 0369
          FMP  ZMX2                  255  1020  39 0624 0774
          STU  A12                   256  0774  21 0478 0431
          RAU  A11                   257  0431  60 0428 0383
          FSB  A12                   258  0383  33 0478 0305
          FMP  AMMM   B              259  0305  39 4100 0500
          FMP  SQRRT                 260  0500  39 0378 0528
          STU  A1     B              261  0528  21 4140 0093
          SXB   0001                 262  0093  53 0001 0349
          NZB  LOPP1       CONT1     263  0349  42 0573 0553
CONT1     LDD  NOLAM                 264  0553  69 0456 0559
          RAS   8001       LOOP9     265  0559  82 8001 0465
LDOP9     RAU  ALPHA                 266  0465  60 0278 0433
          FMP  KAPPA                 267  0433  39 0436 0336
          FMP  KAPPA                 268  0336  39 0436 0386
          STU  ALKAP                 269  0386  21 0190 0193
          FSB  LAMDA  B              270  0193  33 4220 0397
          STU  DIFF                  271  0397  21 0052 0355
          RAU  LAMDA  B              272  0355  60 4220 0925
          FMP  T                     273  0925  39 0358 0458
          LOD              EOOEA     274  0458  69 0361 0000
          STU  ELAMT                 275  0361  21 0366 0469
          RAU  ELAMT                 276  0469  60 0366 0171
          FMP  QOO                   277  0171  39 0324 0824
          FMP  AYEJ   B              278  0824  39 4200 0550
          FMP  ALPHA                 279  0550  39 0278 0578
          FDV  KAY                   280  0578  34 0581 0481
          FDV  DIFF                  281  0481  34 0052 0302
          STU  A3P                   282  0302  21 0506 0609
          RAU  KAPPA                 283  0609  60 0436 0341
          FMP  L                     284  0341  39 0186 0486
          LDD              SINHX     285  0486  69 0339 0319
          STU  SNHKL                 286  0339  21 0044 0447
          FMP  KAPPA                 287  0447  39 0436 0536
          STU  KSHKL                 288  0536  21 0290 0293
          RAU  KAPPA                 289  0293  60 0436 0391
          FMP  L                     290  0391  39 0186 0586
          LDD              COSHX     291  0586  69 0389 0169
          STU  CSHKL                 292  0389  21 0094 0497
          RAU  LAMD,A B              293  0497  60 4220 0975
          BMI  NEG         POS       294  0975  46 0628 0729
NEG       RSU  LAMDA  B              295  0628  61 4220 1025
          FDV  ALPHA                 296  1025  34 0278 0678
          LDD              EODAU     297  0678  69 0531 0450
          STU  SQRLA                 298  0531  21 0636 0439
          FMP  L                     299  0439  39 0186 0686
          LDD              EDOSR     300  0686  69 0489 0369
          STU  SINLL                 301  0489  21 0194 0547
          RAU  SQRLA                 302  0547  60 0636 0441
          FMP  L                     303  0441  39 0186 0736
          LDD              EOOCR     304  0736  69 0539 0269
          STU  COSLL                 305  0539  21 0294 0597
          RSU  SINLL                 306  0597  61 0194 0399
          FMP  SQRLA                 307  0399  39 0636 0786
          FDV  COSLL                 308  0786  34 0294 0344
          FMP  CSHKL                 309  0344  39 0094 0394
          FSB  KSHKL                 310  0394  33 0290 0417
          FMP  A3P                   311  0417  39 0506 0556
          STU  A3     B              312  0556  21 4240 0343
          SXB   0001                 313  0343  53 0001 0449
          NZB  LOOP9       CONT3     314  0449  42 0465 0603
POS       FDV  ALPHA                 315  0729  34 0278 0728
          LDD              EOOAU     316  0728  69 0631 0450
          STU  SQRLA                 317  0631  21 0636 0589
          FMP  L                     318  0589  39 0186 0836
          LDD              SINHX     319  0836  69 0639 0319
          STU  SNHLL                 320  0639  21 0444 0647
          RAU  SQRLA                 321  0647  60 0636 0491
          FMP  L                     322  0491  39 0186 0886
          LDD              COSHX     323  0886  69 0689 0169
          STU  CSHLL                 324  0689  21 0494 0697
          RAU  SNHLL                 325  0697  60 0444 0499
          FDV  CSHLL                 326  0499  34 0494 0544
          FMP  CSHKL                 327  0544  39 0094 0594
          FMP  SQRLA                 328  0594  39 0636 0936
          FSB  KSHKL                 329  0936  33 0290 0467
          FMP  A3P                   330  0467  39 0506 0606
          STU  A3     B              331  0606  21 4240 0393
          SXB   0001                 332  0393  53 0001 0549
          NZB  LODP9       CONT3     333  0549  42 0465 0603
CONT3     RAU  ONE                   334  0603  60 0656 0411
          RSL   8003                 335  0411  66 8003 0519
          STL  N                     336  0519  20 0673 0026
          RAU  ZERO                  337  0026  60 0164 0569
          STU  HOLD        LOOP3     338  0569  21 0920 0723
LOOP3     RAU  N                     339  0723  60 0673 0827
          FAD  TWD                   340  0827  32 0820 0747
          STU  N                     341  0747  21 0673 0176
          FMP  PI                    342  0176  39 0779 0829
          FDV  TWO                   343  0829  34 0820 1070
          FOV  L                     344  1070  34 0186 0986
          STU  ARG1                  345  0986  21 0340 0443
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         FMP  ARG1                 346   0443   39  0340  0390
         FMP  ALPHA                347   0390   39  0278  0778
         STU  ARG2                 348   0778   21  0182  0485
         RAU  ZERO                 349   0485   60  0164  0619
         STU  A24                  350   0619   21  0874  0877
         LDD  NOLAM                351   0877   69  0456  0659
         RAB   8001     LOPS       352   0659   82  8001  0515
LOPS     RAU  ARG2                 353   0515   60  0182  0187
         FAD  LAMDA  B             354   0187   32  4220  0797
         STU  A2S41                355   0797   21  0352  0405
         RAU  ARG2                 356   0405   60  0182  0287
         FAD  ALKAP                357   0287   32  0190  0517
         STU  A2S42                358   0517   21  0272  1075
         RAU  CSHKL                359   1075   60  0094  0599
         FMP  ALPHA                360   0599   39  0278  0828
         FMP  000                  361   0828   39  0324  0924
         FMP  AYEJ   B             362   0924   39  4200  0600
         FDV  KAY                  363   0600   34  0581  0681
         FDV  A2S41                364   0681   34  0352  0402
         FDV  A2S42                365   0402   34  0272  0322
         FAD  A24                  366   0322   32  0874  0601
         STU  A24                  367   0601   21  0874  0927
         SXB   0001                368   0927   53  0001  0483
         NZB  LOPS      CONT9      369   0483   42  0515  0337
CONT9    RAU  ZERO                 370   0337   60  0164  0659
         STD  SETT                 371   0669   24  0372  1125
         LDD  INDXM                372   1125   69  0076  0879
         RAB   8001     LOPP2      373   0879   82  8001  0535
LOPP2    RAU  ARG2                 374   0535   60  0182  0387
         FMP  ARG2                 375   0387   39  0182  0282
         STU  ARGSQ                376   0282   21  1036  0739
         RAU  STAA   8             377   0739   60  4120  1175
         FMP  STAA   8             378   1175   39  4120  1120
         FAD  ARGSQ                379   1120   32  1036  0413
         STU  DENOM                380   0413   21  0268  0271
         RAU  AMMM   8             381   0271   60  4100  0455
         FMP  ARG2                 382   0455   39  0182  0332
         FDV  DENOM                383   0332   34  0268  0318
         FAD  SETT                 384   0318   32  0372  0649
         STU  SETT                 385   0649   21  0372  1225
         SXB   0001                386   1225   53  0001  0731
         NZB  LOPP2     CONT4      387   0731   42  0535  0585
CONT4    FAD  A24                  388   0585   32  0874  0651
         STU  A2SUM                389   0651   21  0706  0709
         RAU  ARG2                 390   0709   60  0182  0437
         FMP  T                    391   0437   39  0358  0508
         FSB  FIFTY                392   0508   33  0461  0487
         BMI  GO        STO        393   0487   46  0440  0541
STO      LDD  ZERO                 394   0541   69  0164  0567
         STD  TERM                 395   0567   24  1170  0773
         RAU  TERM                 396   0773   60  1170  1275
         FAD  HOLD                 397   1275   32  0920  0847
         STU  HOLD                 398   0847   21  0920  0823
         NZU            CONTS      399   0823   44  0977  0878
         RAU  HOLD                 400   0977   60  0920  1325
         FSB  TERM      ADD        401   1325   33  1170  0897
GO       RSU  ARG2                 402   0440   61  0182  0537
         FMP  T                    403   0537   39  0358  0558
         LDD            EOOEA      404   0558   69  0511  0000
         STU  EAG2T                405   0511   21  0416  0719
         RAU  L                    406   0719   60  0186  0591
         FMP  L                    407   0591   39  0186  1086
         FDV  ALPHA                408   1086   34  0278  0928
         FDV  N                    409   0928   34  0673  0873
         FDV  PI                   410   0873   34  0779  0929
         STU  A2DIV                411   0929   21  0184  0587
         RAU  EAG2T                412   0587   60  0416  0321
         FMP  ARG1                 413   0321   39  0340  0490
         FDV  A2DIV                414   0490   34  0184  0284
         STU  A2DDT                415   0284   21  0338  0641
         FMP  A2SUM                416   0641   39  0706  0756
         STU  TERM      ADD        417   0756   21  1170  0897
ADD      RAU  HOLD                 418   0897   60  0920  1375
         FAD  TERM                 419   1375   32  1170  0947
         STU  HOLD                 420   0947   21  0920  0923
         RAU  TERM                 421   0923   60  1170  1425
         FDV  HOLD                 422   1425   34  0920  1220
         RAM   8003                423   1220   67  8003  1027
         RAU   8002                424   1027   60  8002  0635
         FSB  CRIT                 425   0635   33  0388  0565
         BMI  CONTS     LOOP3      426   0565   46  0878  0723
CONTS    LDD  INDXM                427   0878   69  0076  0979
         RAB   8001                428   0979   82  8001  0685
         RAU  ZERO                 429   0685   60  0164  0769
         STU  CELL      LOP        430   0769   21  0974  1077
LOP      RAU  A1     B             431   1077   60  4140  0495
         FAD  CELL                 432   0495   32  0974  0701
         STU  CELL                 433   0701   21  0974  1127
         SXB   0001                434   1127   53  0001  0533
         NZB  LOP       GOON1      435   0533   42  1077  0637
GOON1    LDD  NOLAM                436   0637   69  0456  0759
         RAB   8001                437   0759   82  8001  0615
         RAU  ZERO                 438   0615   60  0164  0819
         STU  CELL3     LOP2       439   0819   21  1024  1177
LOP2     RAU  A3     B             440   1177   60  4240  0545
         FAD  CELL3                441   0545   32  1024  0751
         STU  CELL3                442   0751   21  1024  1227
         SXB   0001                443   1227   53  0001  0583
         NZB  LOP2      GOON2      444   0583   42  1177  0687
GOON2    RAU  HOLD                 445   0687   60  0920  1475
         FAD  CELL                 446   1475   32  0974  0801
         FAD  CELL3                447   0801   32  1024  0851
         FMP  KAY                  448   0851   39  0581  0781
         RSL   8003                449   0781   66  8003  0789
         STL  ANSWR     FINIS      450   0789   20  0493  0396
FINIS    LDD  ANSWR                451   0396   69  0493  0446
         STD   1977                452   0446   24  1977  0430
         LDD  CELL                 453   0430   69  0974  1277
         STD   1978                454   1277   24  1978  0831
         LDD  HOLD                 455   0831   69  0920  0973
         STD   1979                456   0973   24  1979  0382
         LDD  CELL3                457   0382   69  1024  1327
         STD   1980                458   1327   24  1980  0633
         LDD  X                    459   0633   69  1136  0839
         STD   1981                460   0839   24  1981  0334
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	LDD			461	0334	69	0358	0561
	STD	1982		462	0561	24	1982	0735
	PCH	1977	START	463	0735	71	1977	1999
ONE	10	0000	0051	464	0656	10	0000	0051
TWO	20	0000	0051	465	0820	20	0000	0051
PI	31	4159	2751	466	0779	31	4159	2751
FIFTY	50	0000	0052	467	0461	50	0000	0052
ZERO	00	0000	0000	468	0164	00	0000	0000
CRIT	10	0000	0047	469	0388	10	0000	0047
ARTAN	STO	EXIT		4	1800	24	1803	1806
	NZE		EXIT	5	1806	45	1810	1803
	BMI	MINUS		6	1810	46	1813	1814
	STU	ARTAO		7	1814	21	1818	1821
	LDD	FPONE		8	1821	69	1824	1827
	STD	ENNNN		9	1827	24	1830	1833
	STO	AYE	SUBTR	10	1833	24	1836	1839
MINUS	RSL	8003		11	1813	66	8003	1871
	STL	ARTAO		12	1871	20	1818	1822
	RSU	FPONE		13	1822	61	1824	1829
	STO	ENNNN		14	1829	24	1830	1883
	STU	AYE	SUBTR	15	1883	21	1836	1839
SUBTR	RAU	ARTAO		16	1839	60	1818	1823
	FSB	FPONE		17	1823	33	1824	1801
	NZE	DIFFE		18	1801	45	1804	1805
	LDD	PIOV4		19	1805	69	1808	1811
	STO	FUNGT	MULTA	20	1811	24	1864	1817
DIFFE	BMI	SMALL		21	1804	46	1807	1858
	FSB	FPONE		22	1858	33	1824	1851
	BMI	NEGAT	POSIT	23	1851	46	1854	1855
NEGAT	LDD	PIOV4		24	1854	69	1808	1861
	STD	FUNGT		25	1861	24	1864	1867
	RAU	ARTAO		26	1867	60	1818	1873
	FAD	FPONE		27	1873	32	1824	1802
	STU	TURRN		28	1802	21	1856	1809
	FSB	FPTWO		29	1809	33	1812	1889
	FDV	TURRN	COMBI	30	1889	34	1856	1857
SMALL	FAD	ARTAO		31	1807	32	1818	1845
	BMI		NEGAT	32	1845	46	1848	1854
	RAU	ARTAO		33	1848	60	1818	1874
	FSB	LOSNO		34	1874	33	1877	1853
	BMI	TINEY		35	1853	46	1859	1860
	RAU	ARTAO		36	1860	60	1818	1825
	STL	FUNGT	COMBI	37	1825	20	1864	1857
POSIT	LDD	PIOV2		38	1855	69	1862	1815
	STO	FUNGT		39	1815	24	1864	1868
	FSB	UPSNO		40	1868	33	1872	1849
	BMI		MULTA	41	1849	46	1852	1817
	RSU	FPONE		42	1852	61	1824	1879
	FDV	ARTAO	COMBI	43	1879	34	1818	1857
COMBI	STU	TURRN		44	1857	21	1856	1863
	STO	TURRM		45	1863	24	1816	1819
	FMP	TURRN		46	1819	39	1856	1865
	STU	ARGUE	FIGUR	47	1865	21	1820	1875
FIGUR	RAU	TURRM		48	1875	60	1816	1826
	FAD	FUNGT		49	1826	32	1864	1841
	STU	FUNGT		50	1841	21	1864	1869
	RAU	ENNNN		51	1869	60	1830	1835
	FAD	FPTWO		52	1835	32	1812	1840
	STU	ENNNN		53	1840	21	1830	1834
	RSU	TURRN		54	1834	61	1856	1866
	FMP	ARGUE		55	1866	39	1820	1870
	STU	TURRN		56	1870	21	1856	1876
	FDV	ENNNN		57	1876	34	1830	1880
	STU	TURRM		58	1880	21	1816	1828
	RAM	FUNGT		59	1828	67	1864	1878
	STL	FMAGG		60	1878	20	1884	1837
	RAM	TURRM		61	1837	67	1816	1831
	RAU	8002		62	1831	60	8002	1890
	FDV	FMAGG		63	1890	34	1884	1885
	FSB	SIZES		64	1885	33	1838	1881
	BMI	MULTA	FIGUR	65	1881	46	1817	1875
TINEY	RAU	ARTAO	MULTA	66	1859	60	1818	1817
MULTA	RAU	AYE		67	1817	60	1836	1891
	FMP	FUNGT	EXIT	68	1891	39	1864	1803
FPONE	10	0000	0051	69	1824	10	0000	0051
FPTWO	20	0000	0051	70	1812	20	0000	0051
SIZES	10	0000	0043	71	1838	10	0000	0043
PIOV2	15	7079	6351	72	1862	15	7079	6351
PIOV4	78	5398	1650	73	1808	78	5398	1650
UPSNU	10	0000	0060	74	1872	10	0000	0060
LOSNU	10	0000	0040	75	1877	10	0000	0040

AN INVESTIGATION OF NUCLEAR EXCURSIONS TO
DETERMINE THE SELF-SHUTDOWN EFFECTS IN
THERMAL, HETEROGENEOUS, HIGHLY ENRICHED
LIQUID-MODERATED REACTORS

by

JOHN ROBERT FAGAN

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The safe operation of nuclear reactors is imperative if there is to be increased engineering application of these systems. Transient reactor experiments, such as the SPERT tests, have demonstrated that thermal, heterogeneous, liquid-moderated reactor systems will safely shut themselves down following step and ramp insertions of limited amounts of excess reactivity. It is important that a model based on the nuclear, thermodynamic and hydrodynamic properties of the reactor system be developed to explain this phenomena so that it can be used in the design of new systems.

Equations for the fine structure of the temperature distribution in a unit cell of a heterogeneous reactor during a transient burst were derived based on the known power and fuel surface temperature distributions. A model based on recognized shutdown effects was developed to calculate the excess reactivity during a transient using the temperature distributions to define the deposition of energy. The calculated excess reactivities show this model to be satisfactory. The effect on reactivity due to steam formation required one empirical parameter which can probably be removed when a greater knowledge of transient boiling is available.